Dr. Ahmed Soil Mechanics Notes Chapter Six (Shear Strength of Soil)

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1 author:

Ahmed Al-Obaidi
Tikrit University

CHAPTER SIX

SHEAR STRENGTH

OF SOIL

Lecture Notes

Soil

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3rd Class

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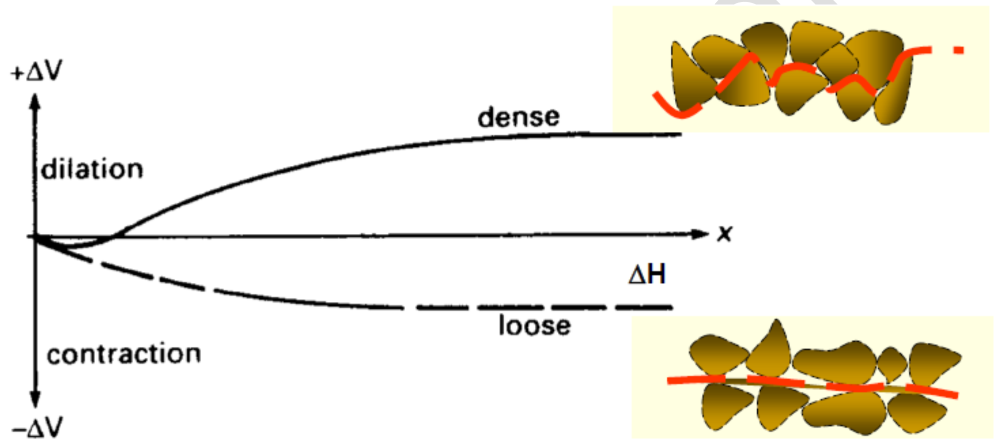


Assistant Prof.

Dr. Ahmed Al-Obaidi

CHAPTER SIX

SHEAR STRENGTH OF SOIL

6.1 Introduction

The strength of material is the greatest stress it can sustain.

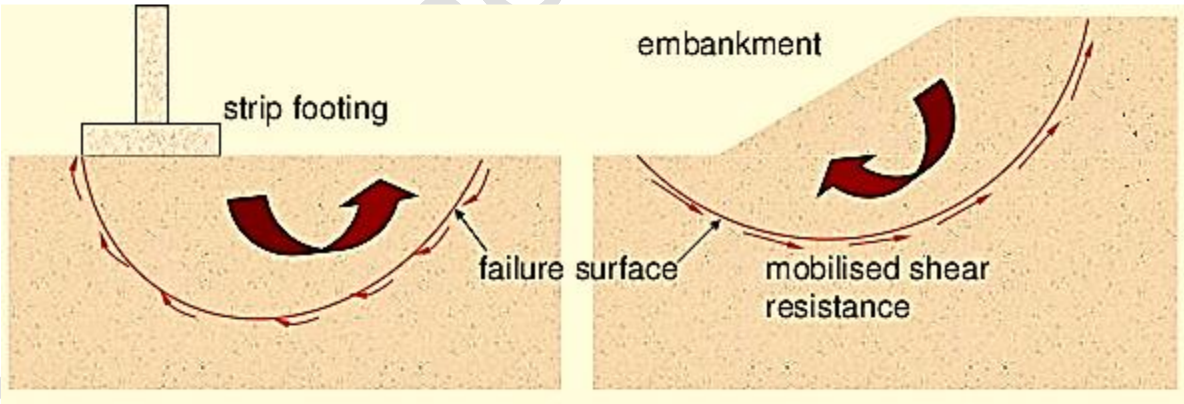
The safety of any geotechnical structure is dependent on the strength of the soil.

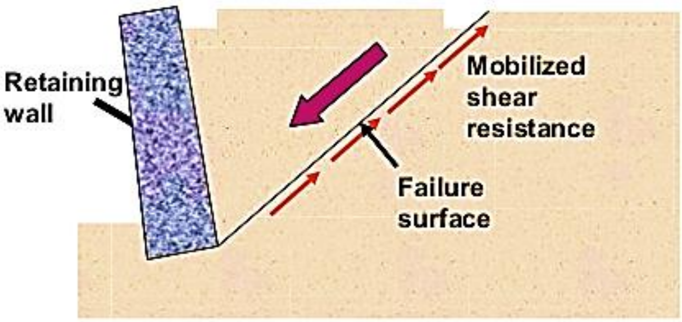
Shearing resistance is important to analyze soil stability problems such as; Bearing capacity, Slope stability, Lateral earth pressure on earth-retaining structures, and Pavement.

The shear strength of soil is its resistance to shearing stresses.

It is a measure of the soil resistance to deformation by continuous displacement of its individual soil particles.

The shear strength of a soil mass is the internal resistance per unit area that the soil mass can offer to resist failure and to slide along any plane inside it.





6.2 Mohr–Coulomb Failure Criterion

Coulomb, 1776 considered that failure of soil is a curved line.

$$\tau_f = f(\sigma)$$

Mohr (1900) presented a theory for rupture in materials that contended that a material fails because of a critical combination of normal stress and shearing stress. Thus, the failure can be expressed in the following form:

$$\tau_f = c + \sigma \tan\phi$$

Where: c = cohesion

ϕ = angle of internal friction

σ = normal stress on the failure plane

τ_f = shear strength

The preceding equation is called the Mohr–Coulomb failure criterion.

In saturated soil, $\sigma = \sigma' + u$

The Mohr–Coulomb failure criterion, in terms of effective stress, will be:

$$\tau_f = c' + \sigma' \tan\phi'$$

Where c' = cohesion and ϕ' = friction angle, based on effective stress.

So that soil derives its shear strength from two sources:

Cohesion between particles (stress independent component), Cementation between sand grains, Electrostatic attraction between clay particles

Frictional resistance between particles (stress dependent component)

The value of (c) for sand and inorganic silt = 0.

For normally consolidated clays, (c) can be = 0.

Overconsolidated Clays have values of (c) that are greater than 0.

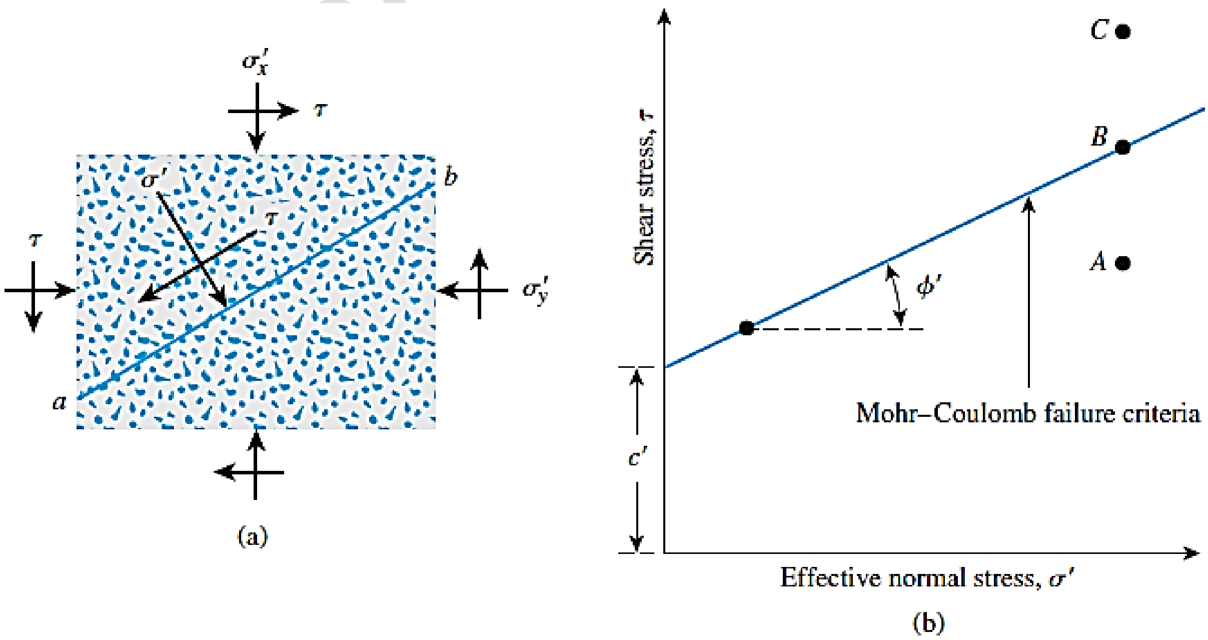
The angle of friction, ϕ' is sometimes referred to as the drained angle of friction.

Typical values of ϕ' for some granular soils are given in Table.

Typical Values of Drained Angle of Friction for Sands and Silts

Soil type	ϕ' (deg)
Sand: Rounded grains	
Loose	27–30
Medium	30–35
Dense	35–38
Sand: Angular grains	
Loose	30–35
Medium	35–40
Dense	40–45
Gravel with some sand	34–48
Silts	26–35

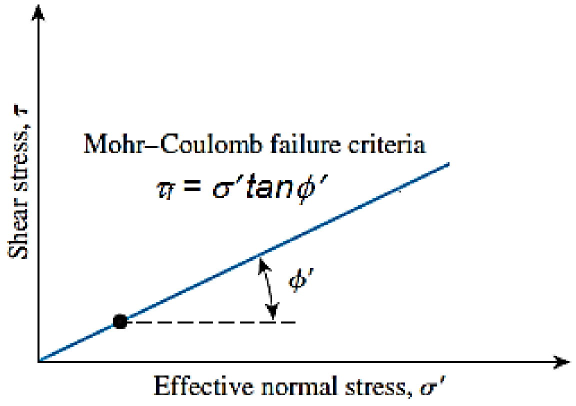
From the figure below, which shows an elemental soil mass. Let the effective normal stress and the shear stress on the plane (ab) be (σ') and (τ), respectively. If the magnitudes of σ' and τ on the plane (ab) are such that they plot as point A, shear failure will not occur along the plane. If the effective normal stress and the shear stress on plane ab plot as point B (which falls on the failure envelope), shear failure will occur along that plane. A state of stress on a plane represented by point C cannot exist because it plots above the failure envelope, and shear failure in soil would have occurred already.

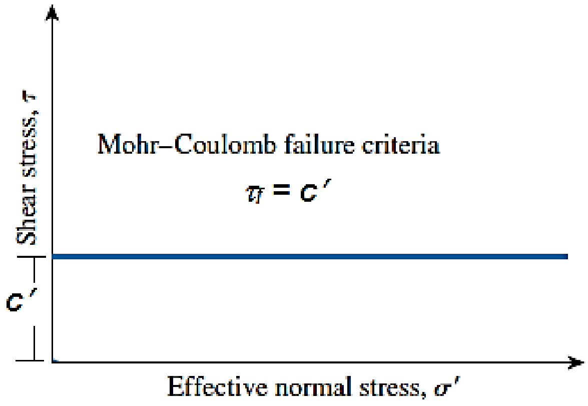


Mohr–Coulomb failure criterion

For cohesionless soils (c) = 0, the failure criterion will be: $\tau_f = \sigma' \tan\phi'$

For saturated soil under undrained condition, the failure criterion will be: $\tau_f = c'$





Example (1)

What is the shearing strength of soil along a horizontal plane at a depth of 4 m in a deposit of sand having the following properties:

Angle of internal friction, $\phi = 35°$, Dry unit weight, $\gamma_d = 17$ kN/m^3, Specific gravity, Gs = 2.7. Assume the ground water table is at a depth of 2.5 m from the ground surface. Also, find the change in shear strength when the water table rises to the ground surface.

Solution

The effective vertical stress at the plane of interest is

$$\sigma' = 2.50 \times \gamma_d + 1.50 \times \gamma_b$$

Given $\gamma_d = 17$ kN/m^3 and $G_s = 2.7$

We have $\gamma_d = 17 = \dfrac{G_s}{1+e}\gamma_w = \dfrac{2.7}{1+e} \times 9.81$

or $17e = 26.5 - 17 = 9.49$ or $e = \dfrac{9.49}{17} = 0.56$

Therefore, $\gamma_b = \dfrac{G_s - 1}{1+e}\gamma_w = \dfrac{2.7 - 1.0}{1+0.56} \times 9.81 = 10.7$ kN/m^3

Hence $\sigma' = 2.5 \times 17 + 1.5 \times 10.7 = 58.55$ kN/m^2

Hence, the shearing strength of the sand is

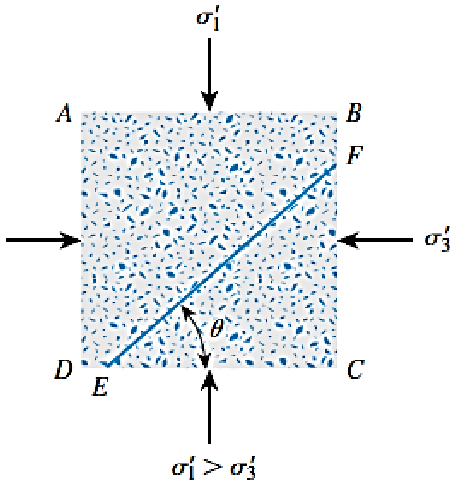
$$\tau_f = \sigma' \tan\phi = 58.55 \times \tan 35° = 41 \text{ kN/m}^2$$

If the water table rises to the ground surface, by a height of 2.5 m, the change in the effective stress will be:

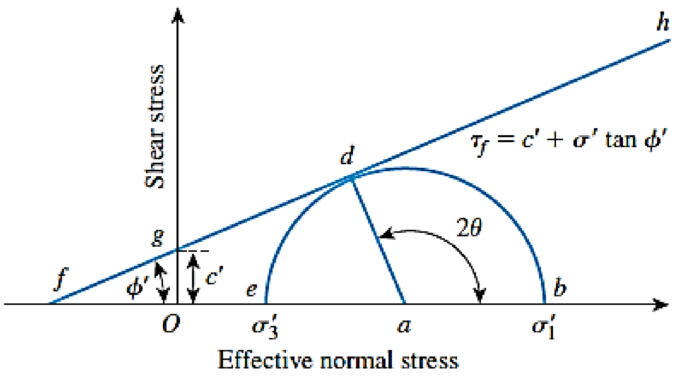
$$\Delta\sigma' = \gamma_d \times 2.5 - \gamma_b \times 2.5 = 17 \times 2.5 - 10.7 \times 2.5 = 15.75 \text{ kN/m}^2 \text{ (negative)}$$

Hence the decrease in shear strength will be,

$$= \Delta\sigma' \tan 35° = 15.75 \times 0.70 = 11 \text{ kN/m}^2$$



Inclination of failure plane
in soil with major principal plane



Mohr's circle and failure envelope

To determine the inclination of the failure plane with the major principal plane, refer to Figure above, where σ_1' and σ_3' are the major and minor effective principal stresses. The failure plane EF makes an angle θ with the major principal plane. To determine the angle θ and the relationship between $\sigma_1'$and σ'_3, which is a plot of the Mohr's circle for the state of stress. In Figure, fgh is the failure envelope defined by the relationship $\tau_f = c + \sigma' \tan\phi'$.The radial line ab defines the major principal plane (CD in Figure), and the radial line ad defines the failure plane (EF in Figure).

It can be shown that $\angle bad = 2\theta = 90 + \phi'$, or

$$\theta = 45 + \frac{\phi'}{2} \qquad (1)$$

Again, from Figure $\dfrac{\overline{ad}}{\overline{fa}} = \sin\phi' \qquad (2)$

$$\overline{fa} = fO + Oa = c' \cot\phi' + \frac{\sigma_1' - \sigma_3'}{2} \qquad (3)$$

Also, $\overline{ad} = \dfrac{\sigma_1' + \sigma_3'}{2}$ (4)

Substituting Eqs. (3) and (4) into Eq. (2) , we obtain

$$\sin \phi' = \frac{\dfrac{\sigma_1' - \sigma_3'}{2}}{c' \cot \phi' + \dfrac{\sigma_1' + \sigma_3'}{2}}$$

or $$\sigma_1' = \sigma_3'\left(\frac{1 + \sin \phi'}{1 - \sin \phi'}\right) + 2c'\left(\frac{\cos \phi'}{1 - \sin \phi'}\right)$$

$$\frac{1 + \sin \phi'}{1 - \sin \phi'} = \tan^2\left(45 + \frac{\phi'}{2}\right)$$

$$\frac{\cos \phi'}{1 - \sin \phi'} = \tan\left(45 + \frac{\phi'}{2}\right)$$

$$\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c' \tan\left(45 + \frac{\phi'}{2}\right)$$

In total stress terms

$$\sigma_1 = \sigma_3 \tan^2\left(45 + \frac{\phi}{2}\right) + 2c \tan\left(45 + \frac{\phi}{2}\right)$$

6.3 Laboratory Test to Determine Shear Strength Parameters

There are several laboratory methods available to determine the shear strength parameters, (c,ϕ, c', ϕ') in the laboratory. They are as follows:

- Direct shear test
- Triaxial test
- Direct simple shear test
- Plane strain triaxial test
- Torsional ring shear test
- Field method: Vane shear test or by any other indirect methods

Test	Isotropic compression	Confined compression (oedometer)	Triaxial compression	Direct shear
Basic conditions	$\sigma_1 = \sigma_3$	No horizontal movement	$\Delta\sigma_a$, σ_c; σ_c constant as $\Delta\sigma_a$ applied	N, T; N constant as T applied
Type of deformation	Volumetric	Primarily volumetric but some distortion	Distortion and volumetric	Primarily distortion, but some volumetric
Stress path	q, p, K_1-line	q, p, K_0-line, β	q, p	q, p, K_0
Uses	For study of purely volumetric strains	Very simple; approximates certain field conditions	Most common test for studying stress-strain and strength properties	Simple test for measuring strength

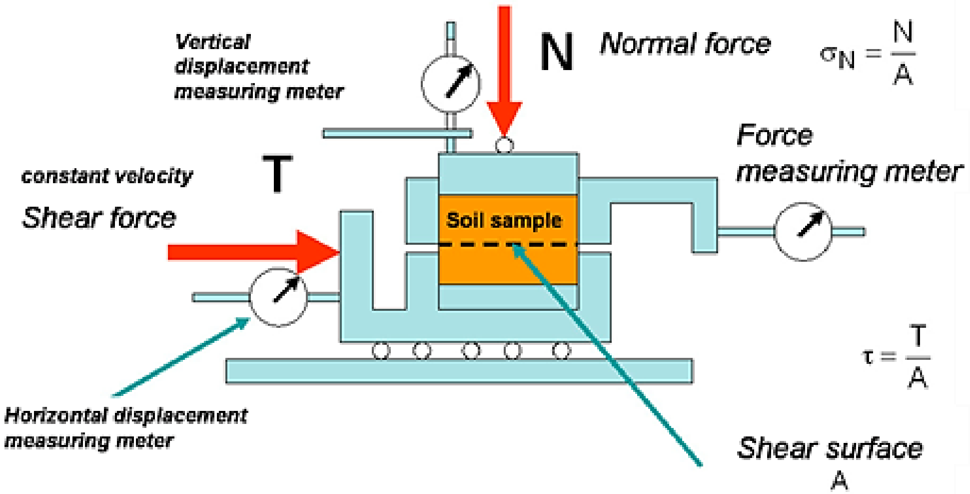
Because soil is a complex material. No one test sufficient for the study of stress-strain behavior. In the above Figure, it can find the common types stress – strain tests.

The direct shear test and the triaxial test are the two commonly used techniques for determining the shear strength parameters.

6.4 Direct Shear Test

The direct shear test is the oldest and simplest form of shear test. The test equipment consists of a metal shear box in which the soil specimen is placed. The soil specimens may be square or circular in plan. The box is split horizontally into halves.

Normal force on the specimen is applied from the top of the shear box. Shear force is applied by moving one-half of the box relative to the other to cause failure in the soil specimen.



For a given test, the normal stress can be calculated as

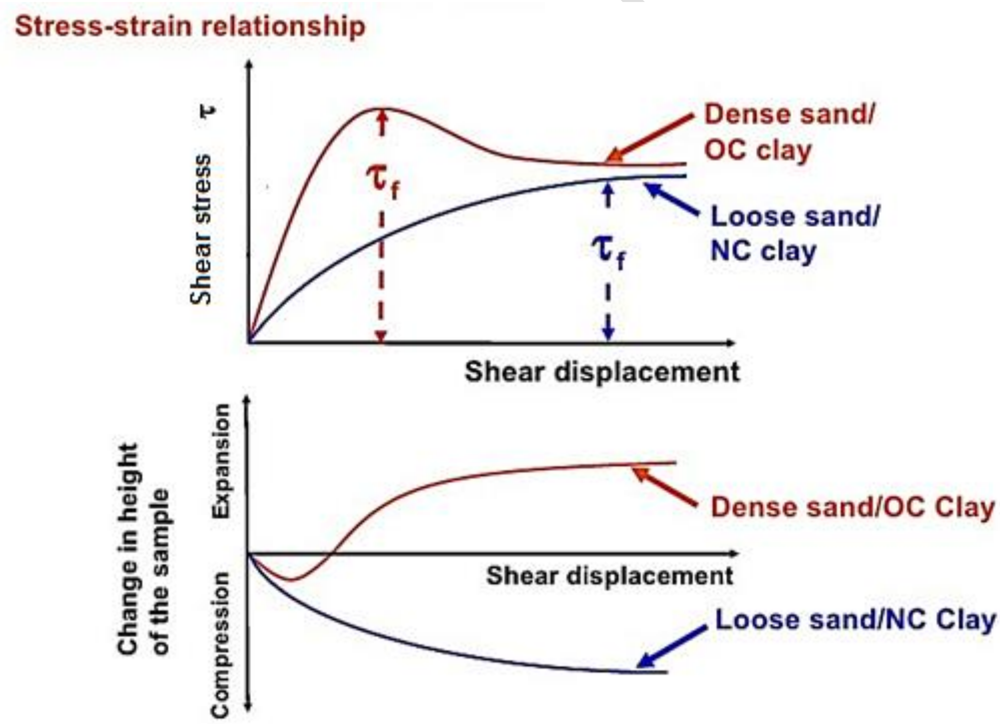
$$\sigma = \text{Normal stress} = \frac{\text{Normal force}}{\text{Cross-sectional area of the specimen}}$$

The resisting shear stress for any shear displacement can be calculated as

$$\tau = \text{Shear stress} = \frac{\text{Resisting shear force}}{\text{Cross-sectional area of the specimen}}$$

The figure shows a typical plot of shear stress and change in the height of the specimen against shear displacement for dry loose and dense sands.

- In loose sand, the resisting shear stress increases with shear displacement until a failure shear stress of (τ_f) is reached. After that, the shear resistance remains approximately constant for any further increase in the shear displacement.
- In dense sand, the resisting shear stress increases with shear displacement until it reaches a failure stress of (τ_f). This (τ_f) is called the peak shear strength. After failure stress is attained, the resisting shear stress gradually decreases as shear displacement increases until it finally reaches a constant value called the ultimate shear strength.



To find the correct internal friction and cohesion of soil, its recommended to carry at least three tests.

The results of this test give normal and shear stress and need to use Mohr's circle or analytic to find major and minor principal stresses, σ_1 and σ_3.

Example (2)

Direct shear tests were performed on a dry sandy soil. The size of the specimen was 50 mm * 50 mm * 19 mm. Test results are as follows:

Test No.	Normal force (N)	Shear force at failure (N)
1	89	53.4
2	133	81.4
3	311	187.3
4	445	267.3

Find the shear strength parameters.

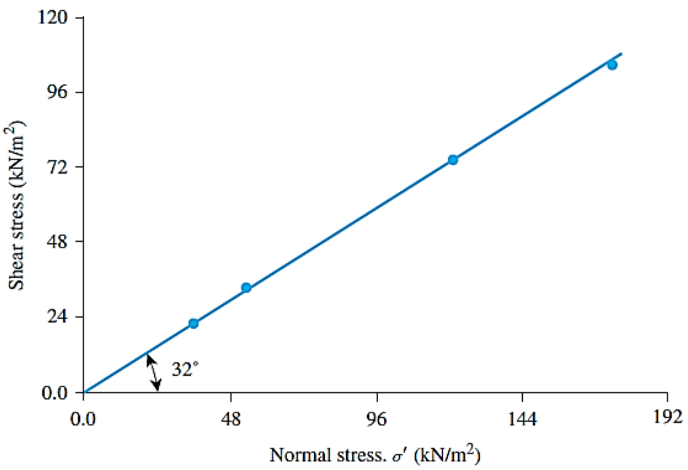
Solution

$$\sigma'\,(\mathrm{kN/m^2}) = \frac{\text{normal force}}{\text{area of specimen}} = \frac{(\text{normal force})}{(1000)(0.05\ \mathrm{m})(0.05\mathrm{m})}$$

$$\tau_f\,(\mathrm{kN/m^2}) = \frac{\text{shear force}}{\text{area of specimen}} = \frac{(\text{shear force})}{(1000)(0.05\ \mathrm{m})(0.05\ \mathrm{m})}$$

Test No.	Normal stress ($\mathrm{kN/m^2}$) $\sigma' = \sigma$	Shear stress at failure, τ_f ($\mathrm{kN/m^2}$)
1	35.6	21.4
2	53.2	32.6
3	124.4	74.9
4	178	106.9

The shear stresses, τ_f obtained from the tests are plotted against the normal stresses in Figure, from which c = 0 and ϕ = 32°.



Example (3)

Following are the results of four drained direct shear tests on an overconsolidated clay, the size of the specimen was 50 mm * 50 mm * 25 mm Find the shear strength parameters.

Test No.	Normal force (N)	Shear force at failure (N)
1	150	157.5
2	250	199.9
3	350	257.6
4	550	363.4

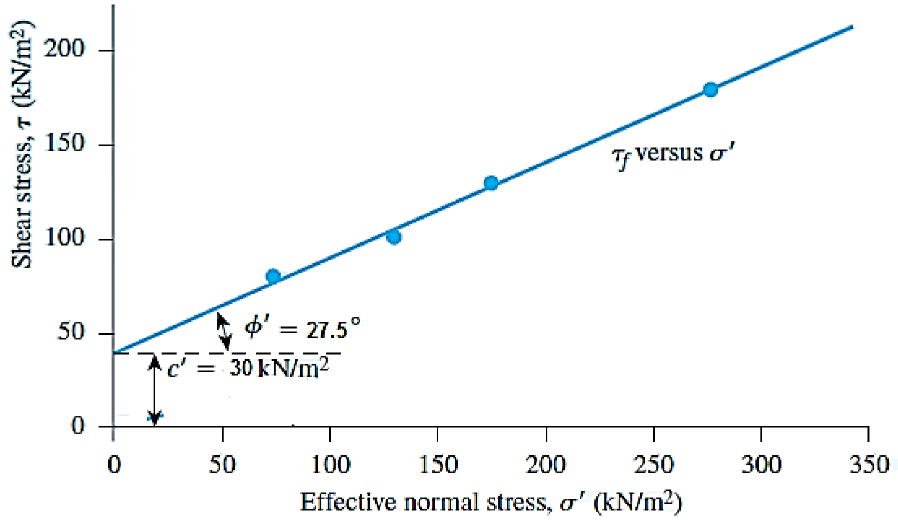
Solution

$$\sigma' \,(\mathrm{kN/m^2}) = \frac{\text{normal force}}{\text{area of specimen}} = \frac{(\text{normal force})}{(1000)(0.05\ \mathrm{m})(0.05\mathrm{m})}$$

$$\tau_f \,(\mathrm{kN/m^2}) = \frac{\text{shear force}}{\text{area of specimen}} = \frac{(\text{shear force})}{(1000)(0.05\ \mathrm{m})(0.05\ \mathrm{m})}$$

Test No.	Normal stress ($\mathrm{kN/m^2}$) $\sigma' = \sigma$	Shear stress at failure, τ_f ($\mathrm{kN/m^2}$)
1	60	63
2	100	80
3	140	103
4	220	145.4

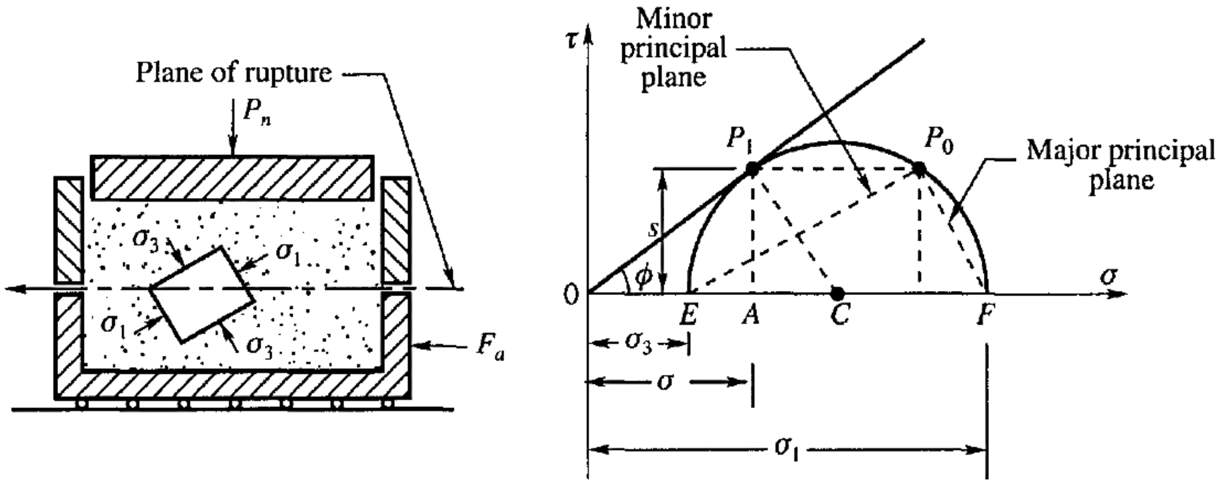
The shear stresses, τ_f obtained from the tests are plotted against the normal stresses in Figure, from which c = 30 kN/m^2 and ϕ = 27.5°



Mohr Diagram for a Direct Shear Test at Failure

In a direct shear test, the sample is sheared along a horizontal plane. This indicates that the failure plane is horizontal.

Point P_I on the stress diagram in Figure represents the stress condition on the failure plane. The coordinates of the point are normal stress = σ and shear stress τ = s.



If it is assumed that the Mohr envelope is a straight line passing through the origin (for cohesionless soil or normally consolidated clays). Therefore the line OP_I must be tangent to the Mohr circle, and the circle may be constructed as follows:

Draw P_IC normal to OP_I, Point C which is the intersection point of the normal with the abscissa is the center of the circle. CP_I is the radius of the circle. The Mohr circle may now be constructed which gives the major and minor principal stresses σ_1 and σ_3 respectively.

Since the failure is on the horizontal plane, the origin of planes P_O may be obtained by drawing a horizontal line through P_I giving P_O. P_OF and P_OE give the directions of the major and minor principal planes respectively.

For analytic solution:

$$\tan\emptyset = \frac{P_IA}{AO}, \qquad P_IO = \frac{P_IA}{\sin\emptyset}, \qquad P_IC = P_IO * \tan\emptyset$$

$$P_I C = (\sigma_1 - \sigma_3)/2$$

$$\sin\emptyset = \frac{AC}{OC} = \frac{(\sigma_1 - \sigma_3)/2}{(\sigma_1 + \sigma_3)/2}, \qquad \frac{(\sigma_1 + \sigma_3)}{2} = \frac{(\sigma_1 - \sigma_3)/2}{\sin\emptyset}$$

in cohesive soil use:

$$\sin\emptyset = \frac{(\sigma_1 - \sigma_3)}{(\sigma_1 + \sigma_3) + 2c\cot\emptyset}$$

Example (4)

A direct shear test was conducted on a remolded sample of sand, gave the following observations at the time of failure: Normal load = 288 N; shear load = 173 N. The cross-sectional area of the specimen = 36 cm^2.

Determine: (a) the angle of internal friction, (b) the magnitude and direction of the principal stresses in the zone of failure.

Solution

Such problems can be solved in two ways, namely graphically and analytically.

(a) Shear stress $\tau = \frac{173}{36} = 4.8 \ \text{N/cm}^2 = 48 \ \text{kN/m}^2$

Normal stress $\sigma = \frac{288}{36} = 8.0 \ \text{N/cm}^2 = 80 \ \text{kN/m}^2$

We know one point on the Mohr envelope. Plot point A with coordinates 48 kN/m^2, and τ_f = 80 kN/m^2. Since cohesion c = 0 for sand, the Mohr envelope OM passes through the origin. The slope of OM gives the angle of internal friction ϕ =31°.

(b) In Figure, draw line AC normal to the envelope OM cutting the abscissa at point C. With C as center and AC as radius, draw Mohr circle C_l which cuts the abscissa at points B and D, which gives

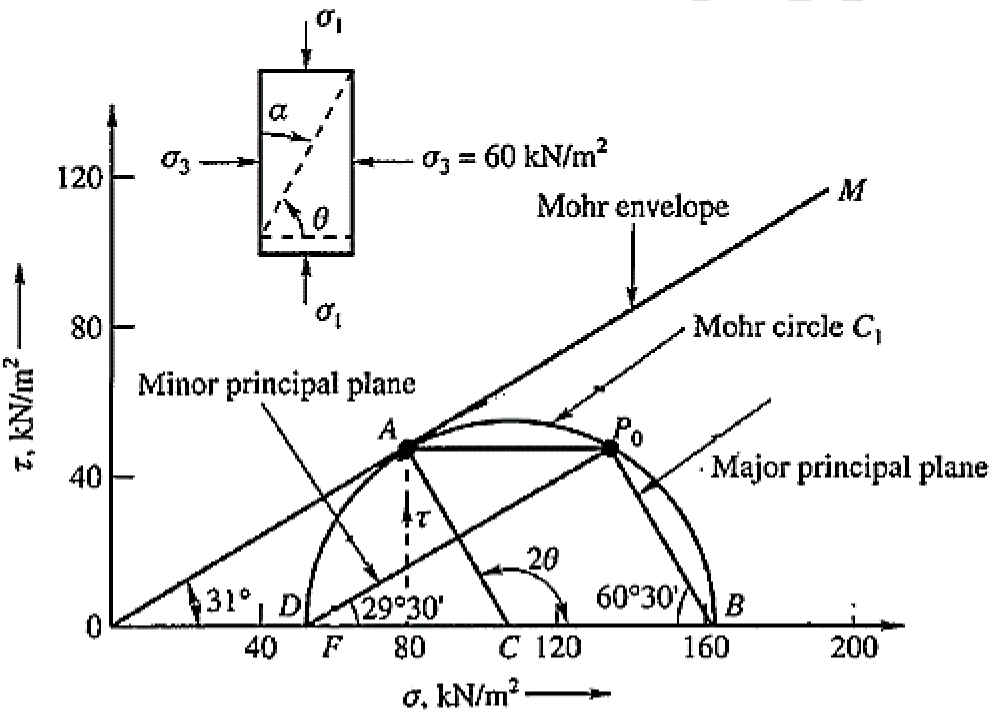
Major principal stress = OB = σ_1 = 163.5 kN/m^2

Minor principal stress = OD = σ_3 = 53.5 kN/m^2

Now, $\angle ACB = 2\alpha$ = twice the angle between the failure plane and the major principal plane. Measurement gives

$2\alpha = 121°$ or $\alpha = 60.5°$

Since in a direct shear test the failure plane is horizontal, the angle made by the major principal plane with the horizontal will be 60.5°. The minor principal plane should be drawn at a right angle to the major principal plane. The directions of the principal planes may also be found by locating the pole P_o which is obtained by drawing a horizontal line from point A which is parallel to the failure plane in the direct shear test. Now P_OB and P_OD give the directions of the major and minor principal planes.



For analytic solution:

$$\tan\emptyset = \frac{48}{80} = 0.6 \rightarrow \emptyset = 31°$$

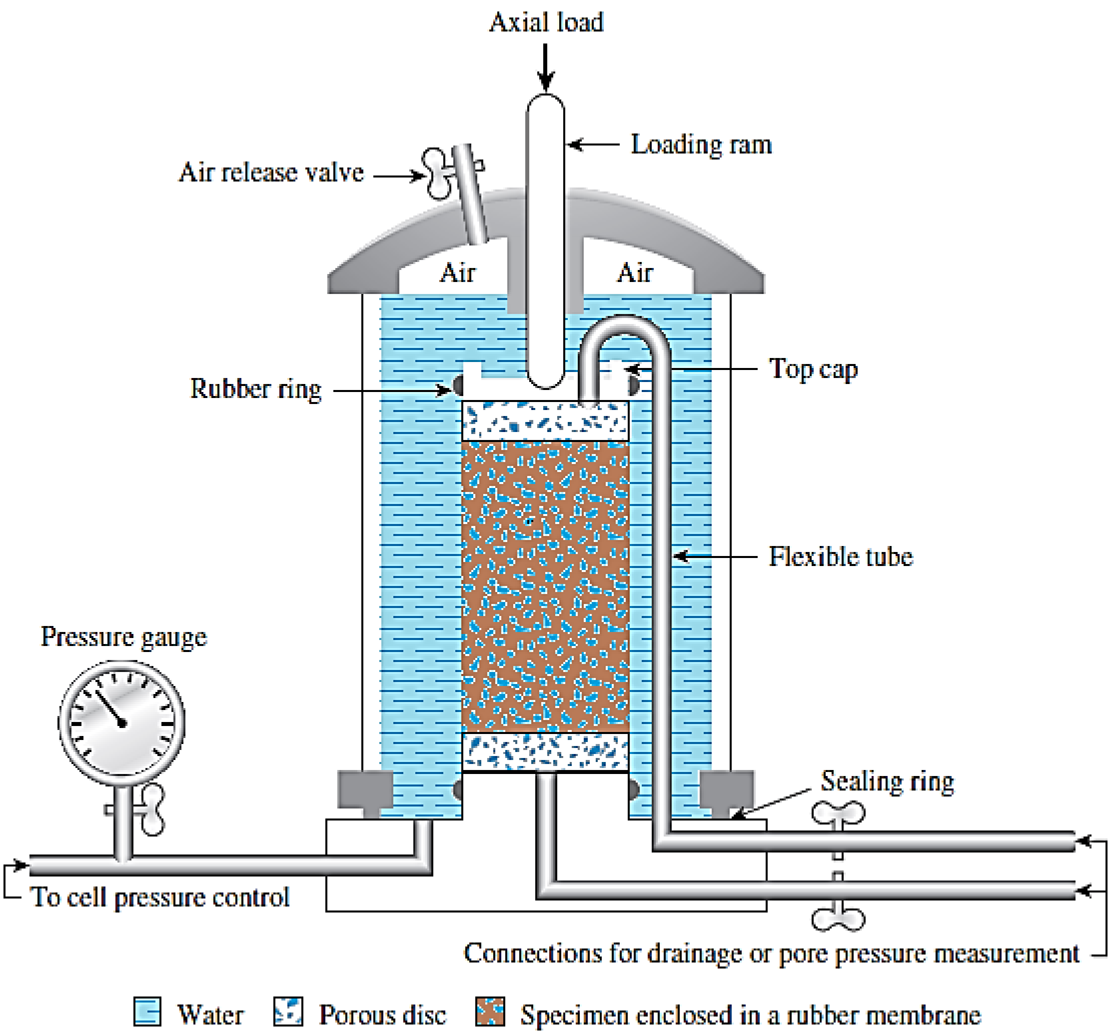
$$AO = \frac{48}{\sin 31} = 93.2\frac{kN}{m^2}, \qquad AC = 93.2 * \tan 31 = 56\frac{kN}{m^2} = (\sigma_1 - \sigma_3)/2$$

$$\sin 31 = \frac{(\sigma_1 - \sigma_3)/2}{(\sigma_1 + \sigma_3)/2}, \qquad \frac{(\sigma_1 + \sigma_3)}{2} = \frac{56}{\sin 31} = 108.73\ kN/m^2$$

$\sigma_1 = 165\ kN/m^2$, $\sigma_3 = 53\ kN/m^2$, $2\theta = 180 - (180 - 31 - 90) = 121$, $\theta = 60.5° = 45+\phi/2$

6.5 Triaxial Shear Test

The triaxial shear test is one of the most reliable methods available for determining shear strength parameters. It is used widely for research and conventional testing. A diagram of the triaxial test layout is shown in Figure.



The figure shows a triaxial test in progress in the laboratory.

In this test, a soil specimen about 36 mm in diameter and 76 mm long. A thin rubber membrane encased the specimen and placed inside a plastic cylindrical chamber that usually is filled with water or glycerin.

The test consists of two-stage the first one is isotropic loading where the specimen is subjected to a confining pressure by compression of the fluid in the chamber. The second stage is shearing to cause shear failure in the specimen, where axial stress is applied (sometimes called deviator stress) through a vertical loading ram.

The axial load applied by the loading ram corresponding to a given axial deformation is measured by a proving ring or load cell attached to the ram.

Connections to measure drainage into or out of the specimen, or to measure pressure in the pore water (as per the test conditions), also are provided. The following three standard types of triaxial tests generally are conducted:

1. Consolidated-drained test or drained test (CD test)
2. Consolidated-undrained test (CU test)
3. Unconsolidated-undrained test or undrained test (UU test)

6.5.1 Consolidated-Drained Triaxial Test

In the CD test, the saturated specimen first is subjected to an all-around confining pressure, σ_3, by compression of the chamber fluid. As confining pressure is applied, the pore water pressure of the specimen increases by u_c (if drainage is prevented). This increase in the pore water pressure can be expressed as a non-dimensional parameter in the form

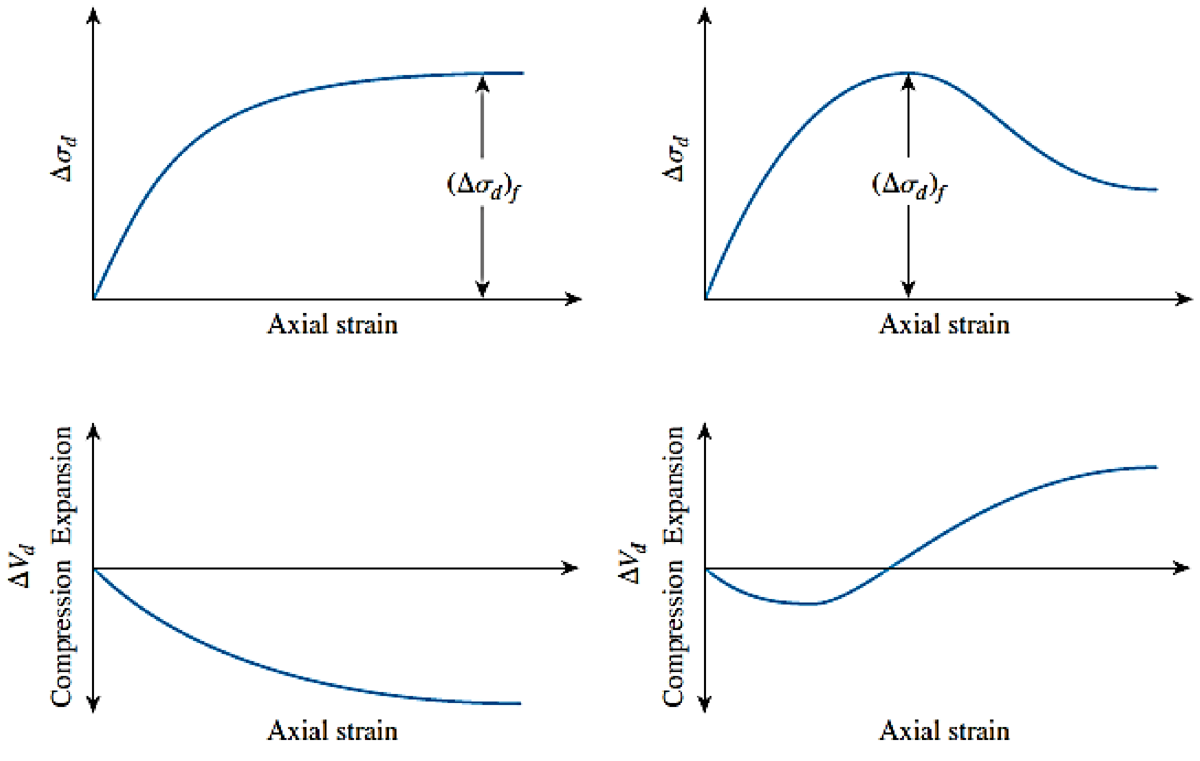
$$B = \frac{u_c}{\sigma_3}$$

where B = Skempton's pore pressure parameter (Skempton, 1954).

Now, if the connection to drainage is opened, dissipation of the excess pore water pressure, and thus consolidation, will occur. With time, u_c will become equal to 0. In saturated soil, the change in the volume of the specimen (ΔV_c)

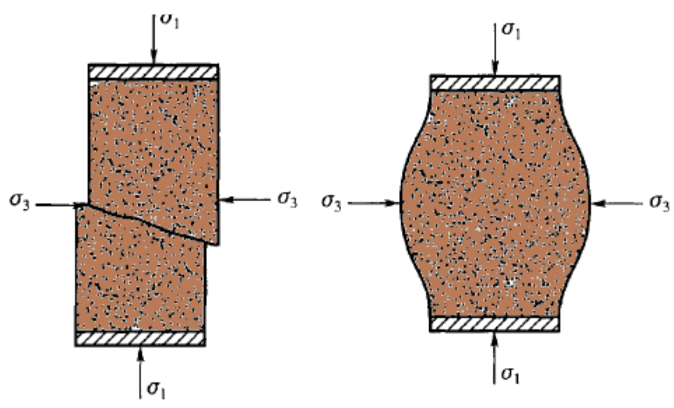
that takes place during consolidation can be obtained from the volume of pore water drained. Next, the deviator stress, σ_d, (= σ_1 - σ_3) on the specimen is increased very slowly. The drainage connection is kept open, and the slow rate of deviator stress application allows complete dissipation of any pore water pressure that developed as a result (Δu_d = 0).

A typical plot of the variation of deviator stress against strain in loose sand and normally consolidated clay also for dense sand and overconsolidated clay is shown in Figure.



loose sand and normally consolidated clay   dense sand and overconsolidated clay

At least three identical samples having the same initial conditions are to be used. Typical shapes of dense and loose sand samples at failure are shown in Figure.



(a) Dense sand   (b) Loose sand

Example (5)

A consolidated drained triaxial test was conducted on a normally consolidated clay. The results are as follows:

- σ_3 = 276 kN/m^2
- $(\Delta\sigma_d)_f$ = 276 kN/m^2

Determine

a. Angle of friction, ϕ'

b. Angle that the failure plane makes with the major principal plane.

c. Find the normal stress σ' and the shear stress τ_f on the failure plane.

b. Determine the effective normal stress on the plane of maximum shear stress.

Solution

For normally consolidated soil, the failure envelope equation is

$\tau_f = \sigma' \tan\phi$ (because c = 0)

a. For the triaxial test, the effective major and minor principal stresses at failure are as follows:

$$\sigma_1 = \sigma_3 + \Delta(\sigma_d)_f = 276 + 276 = 552\ \text{kN/m}^2$$

$$\sigma_3 = 276\ \text{kN/m}^2$$

$$\sin\emptyset = \frac{(\sigma_1 - \sigma_3)/2}{(\sigma_1 + \sigma_3)/2} = \frac{552 - 276}{552 + 276} = 0.333$$

ϕ = 19.45°

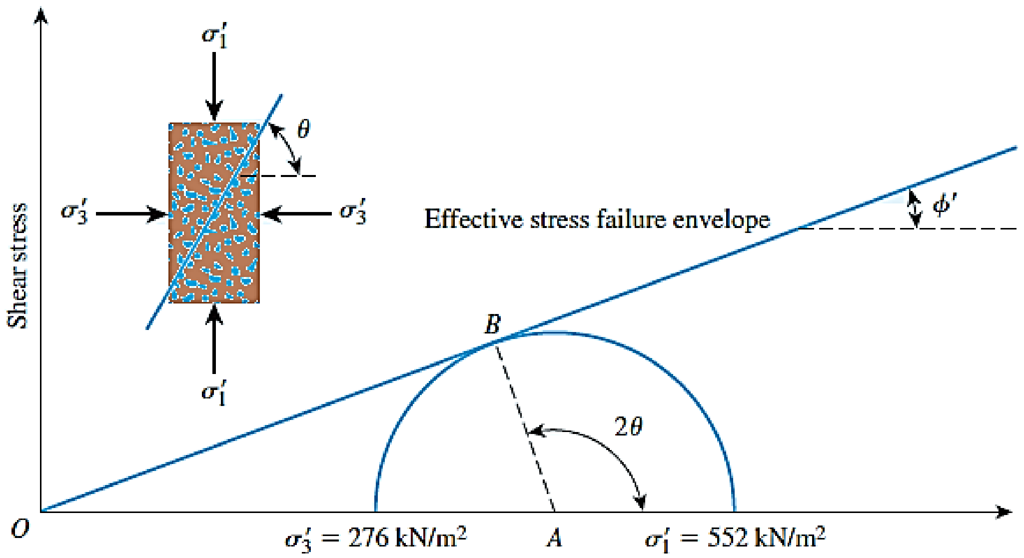
b. $\theta = 45 + {}^{\emptyset}/_{2} = 45 + {}^{19.45}/_{2} = 54.73°$

c. $$\sigma'(\text{on the failure plane}) = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2}\cos 2\theta$$

$$\sigma' = \frac{552 + 276}{2} + \frac{552 - 276}{2}\cos(2 \times 54.73) = \mathbf{368.03\ kN/m^2}$$

$$\tau_f = \frac{\sigma_1' - \sigma_3'}{2}\sin 2\theta$$

$$\tau_f = \frac{552 - 276}{2}\sin(2 \times 54.73) = \mathbf{130.12\ kN/m^2}$$



d. it can be seen that the maximum shear stress will occur on the plane with θ = 45°

$$\sigma' = \frac{\sigma'_1 + \sigma'_3}{2} + \frac{\sigma'_1 - \sigma'_3}{2}\cos 2\theta$$

$$\sigma' = \frac{552 + 276}{2} + \frac{552 - 276}{2}\cos 90 = \mathbf{414\ kN/m^2}$$

<u>Example (6)</u>

The equation of the effective stress failure envelope for normally consolidated clayey soil is $\tau_f = \sigma' \tan 30°$. A drained triaxial test was conducted with the same soil at a chamber confining pressure of 69 kN/m^2. Calculate the deviator stress at failure.

<u>Solution</u>

For normally consolidated clay, c' = 0. Thus,

$$\sigma'_1 = \sigma'_3 \tan^2\left(45 + \frac{\phi'}{2}\right)$$

$$\phi' = 30°$$

$$\sigma'_1 = 69 \tan^2\left(45 + \frac{30}{2}\right) = 207\ \text{kN/m}^2$$

$$(\Delta\sigma_d)_f = \sigma'_1 - \sigma'_3 = 207 - 69 = \mathbf{138\ kN/m^2}$$

Example (7)

The results of two drained triaxial tests on a saturated clay follow:

Specimen I: σ_3 =70 kN/m^2, $(\Delta\sigma_d)_f$ = 130 kN/m^2

Specimen II: σ_3 =160 kN/m^2, $(\Delta\sigma_d)_f$ = 223.5 kN/m^2

Determine the shear strength parameters.

Solution

For Specimen I, the principal stresses at failure are

$\sigma_3 = \sigma'_3$ =70 kN/m^2

$\sigma_1 = \sigma'_1 = \sigma_3 + (\Delta\sigma_d)_f$ =70 + 130 = 200 kN/m^2

Similarly, the principal stresses at failure for Specimen II are

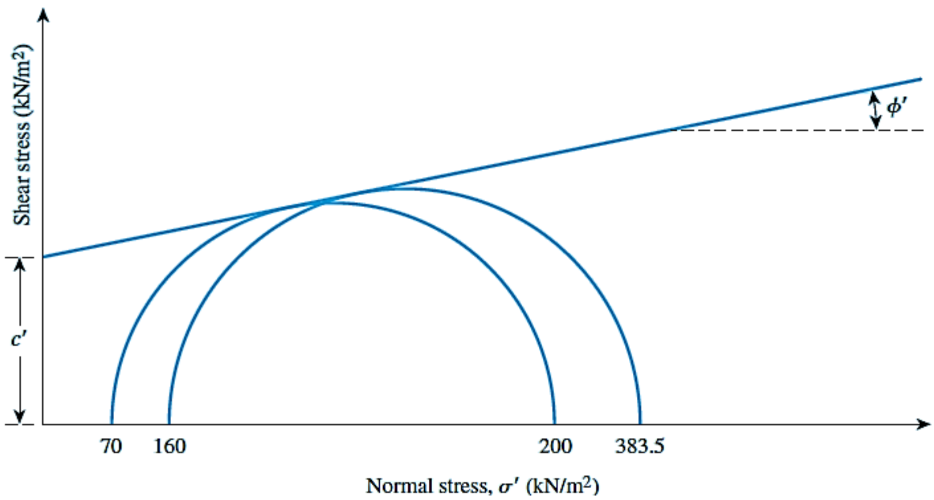
$\sigma_3 = \sigma'_3$ =160 kN/m^2

$\sigma_1 = \sigma'_1 = \sigma_3 + (\Delta\sigma_d)_f$ =160 + 223.5 = 383.5 kN/m^2

$$\phi'_1 = 2\left\{\tan^{-1}\left[\frac{\sigma'_{1(\mathrm{I})} - \sigma'_{1(\mathrm{II})}}{\sigma'_{3(\mathrm{I})} - \sigma'_{3(\mathrm{II})}}\right]^{0.5} - 45°\right\}$$

$$= 2\left\{\tan^{-1}\left[\frac{200 - 383.5}{70 - 160}\right]^{0.5} - 45°\right\} = \mathbf{20°}$$

$$c' = \frac{\sigma'_{1(\mathrm{I})} - \sigma'_{3(\mathrm{I})}\tan^2\left(45 + \frac{\phi'_1}{2}\right)}{2\tan\left(45 + \frac{\phi'_1}{2}\right)} = \frac{200 - 70\tan^2\left(45 + \frac{20}{2}\right)}{2\tan\left(45 + \frac{20}{2}\right)} = \mathbf{20\ kN/m^2}$$



Example (8)

A consolidated drained triaxial test was conducted on a granular soil. At failure $\sigma'_1/\sigma'_3 = 4.0$. The effective minor principal stress at failure was 100 kN/m^2. Compute ϕ' and the principal stress difference at failure.

Solution

$$\sin\phi' = \frac{\sigma'_1/\sigma'_3 - 1}{\sigma'_1/\sigma'_3 + 1} = \frac{4-1}{4+1} = 0.6 \quad \text{or} \quad \phi' = 37^\circ$$

The principal stress difference at failure is

$$(\sigma'_1 - \sigma'_3) = \sigma'_3 \quad \frac{\sigma'_{1f}}{\sigma'_{3f}} - 1 \quad = 100(4-1) = 300 \text{ kN/m}^2$$

Example (9)

A saturated specimen of sand was tested under triaxial drained conditions. The sample failed at a deviator stress of 482 kN/m^2, and the plane of failure made an angle of 60° with the horizontal. Find the magnitudes of the principal stresses. What would be the magnitudes of the deviator stress and the major principal stress at failure for another identical specimen of sand if it is tested at a cell pressure of 200 kN/m^2

Solution

The angle of the failure plane a is expressed as equal to

$$\theta = 45^\circ + \frac{\phi}{2}$$

Since $\theta = 60^\circ$, we have $\phi = 30^\circ$.

$$\sin\phi = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}$$

with $\phi = 30^\circ$, and $\sigma_1 - \sigma_3 = 482$ kN/m^2. Substituting we have

$$\sigma_1 + \sigma_3 = \frac{\sigma_1 - \sigma_3}{\sin\phi} = \frac{482}{\sin 30^\circ} = 964 \text{ kN/m}^2 \qquad \text{(a)}$$

$$\sigma_1 - \sigma_3 = 482 \text{ kN/m}^2 \qquad \text{(b)}$$

solving (a) and (b) we have

$\sigma_1 = 723\ \text{kN/m}^2$, and $\sigma_3 = 241\ \text{kN/m}^2$

For the identical sample

$\phi = 30°$, $\sigma_3 = 200\ \text{kN/m}^2$

$$\sin 30° = \frac{\sigma_1 - 200}{\sigma_1 + 200}$$

Solving for σ_1 we have $\sigma_1 = 600\ \text{kN/m}^2$ and $(\sigma_1 - \sigma_3) = 400\ \text{kN/m}^2$

6.5.2 Consolidated-Undrained Triaxial Test

The consolidated-undrained test is the most common type of triaxial test. In this test, the saturated soil specimen is first consolidated by an all-around chamber fluid pressure, σ'_3, that results in drainage. After the pore water pressure generated by the application of confining pressure is dissipated, the deviator stress, $(\Delta\sigma_d)$, on the specimen, is increased to cause shear failure. During this phase of the test, the drainage line from the specimen is kept closed. Because drainage is not permitted, the pore water pressure, Δu_d, will increase. During the test, simultaneous measurements of $(\Delta\sigma_d)$ and Δu_d are made. The increase in the pore water pressure, Δu_d, can be expressed in a non-dimensional form as:

$$\overline{A} = \frac{\Delta u_d}{\Delta \sigma_d}$$

Where Â Skempton's pore pressure parameter (Skempton, 1954), =AB.

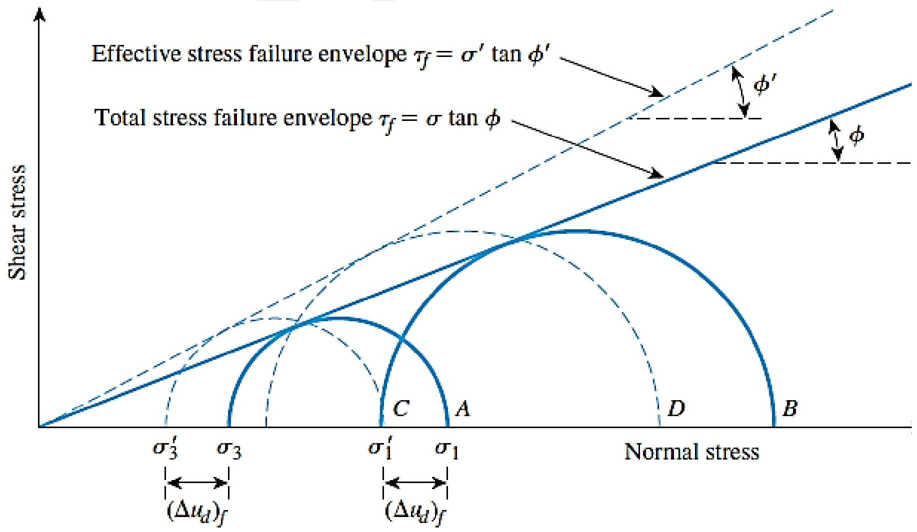
The general patterns of variation of $(\Delta\sigma_d)$ and Δu_d with axial strain for sand and clay soils are shown in Figures. In loose sand and normally consolidated clay, the pore water pressure increases with strain. In dense sand and overconsolidated clay, the pore water pressure increases with a strain to a certain limit, beyond which it decreases and becomes negative (with respect to the atmospheric pressure). This decrease is because of a tendency of the soil to dilate.

Unlike the consolidated drained test, the total and effective principal stresses are not the same in the consolidated-undrained test. Because the pore water pressure at failure is measured in this test, the principal stresses may be analyzed as follows:

- Major principal stress at failure (total): $\sigma_3 + (\Delta\sigma_d)_f = \sigma_1$
- Major principal stress at failure (effective): $\sigma_1 - (\Delta\sigma_d)_f = \sigma'_1$
- Minor principal stress at failure (total): σ_3
- Minor principal stress at failure (effective): $\sigma_3 - (\Delta\sigma_d)_f = \sigma'_3$

In these equations, $(\Delta u_d)_f$ = pressure at failure. The preceding derivations show that: $\sigma_1 - \sigma_3 = \sigma'_1 - \sigma'_3$

Tests on several similar specimens with varying confining pressures may be conducted to determine the shear strength parameters.



Skempton's pore water pressure parameter at failure

$$\Delta u_3 = B\Delta\sigma_3,\ \Delta u_1 = AB(\Delta\sigma_1 - \Delta\sigma_3),\ \text{therefore,}$$

$$\Delta u = \Delta u_1 + \Delta u_3 = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$

$$\text{or}\quad \Delta u = B\Delta\sigma_3 + \bar{A}(\Delta\sigma_1 - \Delta\sigma_3)$$

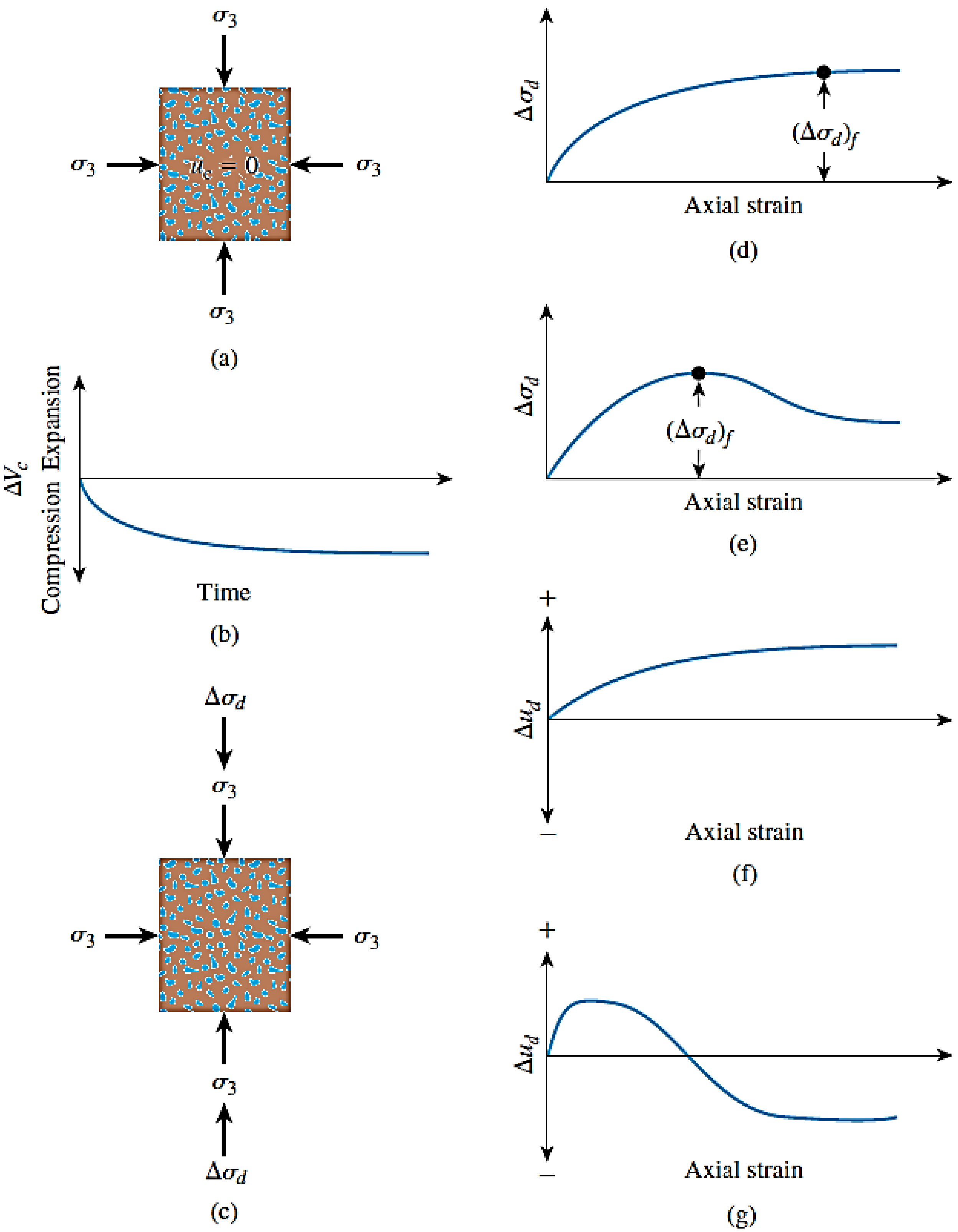
$$\text{where,}\quad \bar{A} = AB$$

for saturated soils $B = 1$, so

$$\Delta u = \Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)$$

The general range of $\hat{A}_f$ values in most clay soils is as follows:

Normally consolidated clays: 0.5 to 1, Overconsolidated Clays: -0.5 to 0



Example (10)

A specimen of saturated sand was consolidated under an all-around pressure of 105 kN/m^2. The axial stress was then increased, and drainage was prevented. The specimen failed when the axial deviator stress reached 70 kN/m^2. The pore water pressure at failure was 50 kN/m^2. Determine

a. Consolidated-undrained angle of shearing resistance, ϕ

b. Drained friction angle, ϕ'

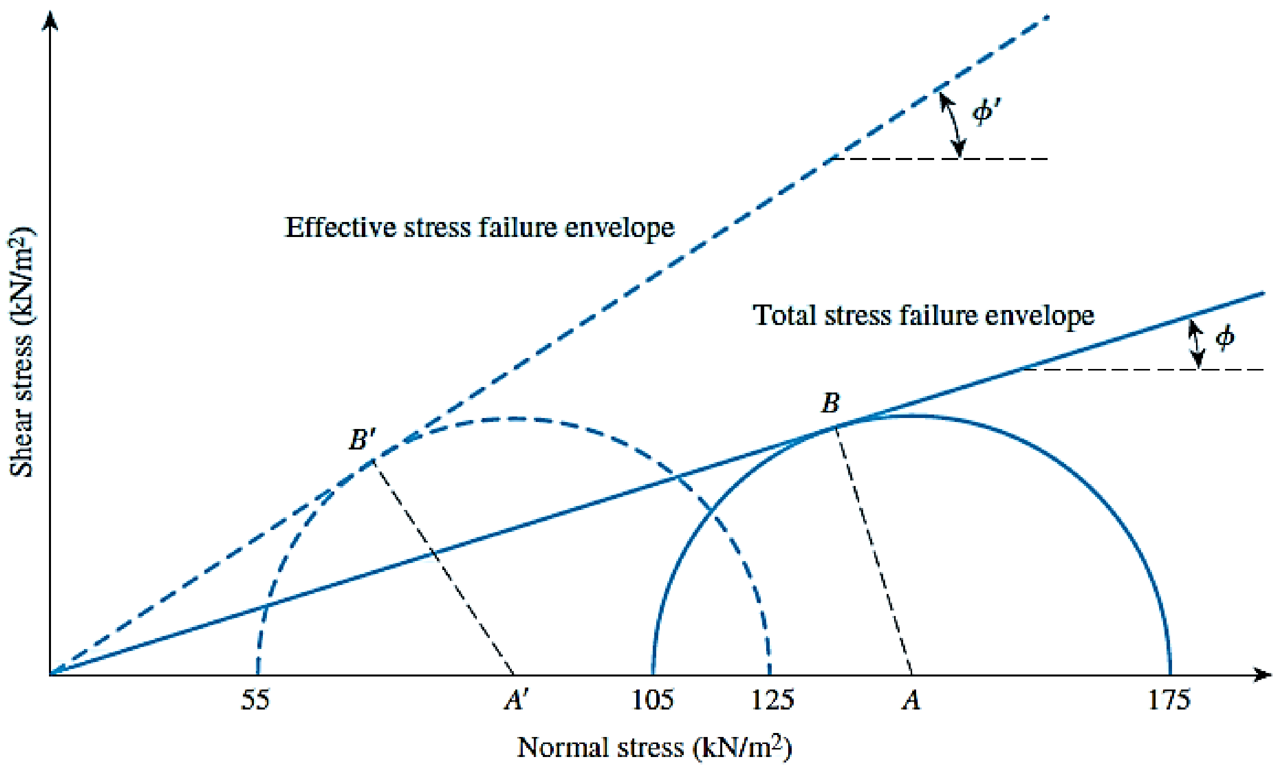
Solution

a. For this case, σ_3 = 105 kN/m^2, σ_1 = 105 + 70 = 175 kN/m^2, and $(\Delta u_d)_f$ = 50 kN/m^2. The total and effective stress failure envelopes are shown in Figure

$$\phi = \sin^{-1}\left(\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}\right) = \sin^{-1}\left(\frac{175 - 105}{175 + 105}\right) \approx \mathbf{14.5°}$$

Part b

$$\phi' = \sin^{-1}\left[\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3 - 2(\Delta u_d)_f}\right] = \sin^{-1}\left[\frac{175 - 105}{175 + 105 - (2)(50)}\right] = \mathbf{22.9°}$$



Example (11)

A normally consolidated clay was consolidated under a stress of 150 kN/m^2, then sheared undrained in axial compression. The principal stress difference at failure was 100 kN/m^2, and the induced pore pressure at failure was 88.5 kN/m^2. Determine:

(a) The Mohr-Coulomb strength parameters, regarding both total and effective stresses analytically

(b) Compute $(\sigma_1/\sigma_3)_f$ and $(\sigma'_1/\sigma'_3)_f$

(c) Determine the theoretical angle of the failure plane in the specimen.

Solution

(a) Given $\sigma_{df} = (\sigma_1 - \sigma_3)_f = 100$ kN/m^2, $\sigma_{3f} = 150$ kN/m^2, $\therefore \sigma_{1f} = 250$ kN/m^2

Effective $\sigma'_{1f} = \sigma_{1f} - u_f = 250 - 88.5 = 161.5$ kN/m^2

$\sigma'_{3f} = \sigma_{3f} - u_f = 150 - 88.5 = 61.5$ kN/m^2

$\sigma_1 = \sigma_3 tan^2\left(45 - \frac{\emptyset}{2}\right) + 2c\tan(45 - \frac{\emptyset}{2})$ c = 0, N.C.C

$\frac{\sigma_1}{\sigma_3} = tan^2\left(45 - \frac{\emptyset}{2}\right) = \frac{1+\sin\emptyset}{1-\sin\emptyset}, or\ sin\emptyset = \frac{\sigma_1-\sigma_3}{\sigma_1+\sigma_3}$

In total stress concept, $\phi = sin^{-1}\frac{100}{250+150} = 0.25,\ \ \emptyset = 14.5°$

In effective stress concept $\emptyset' = sin^{-1}\frac{100}{161.5+61.5} = 0.448,\ \emptyset = 26.64°$

(b) The stress ratio at failure are

$$\frac{\sigma_1}{\sigma_3} = \frac{250}{150} = 1.667,\quad \frac{\sigma'_1}{\sigma'_3} = \frac{161.5}{61.5} = 2.63$$

(c) $\theta = 45 + \phi/2 = 45 + 26.64/2 = 58.32°$

Example (12)

The following results were obtained at failure in a series of consolidated undrained tests, with pore pressure measurement, on specimens of saturated clay. Determine the values of the effective stress parameters c' and ϕ' by drawing Mohr circles.

Test No.	σ_3 (kN/m^2)	$(\sigma_1 - \sigma_3)$ (kN/m^2)	u_f (kN/m^2)
1	150	192	80
2	300	341	154
3	450	504	222

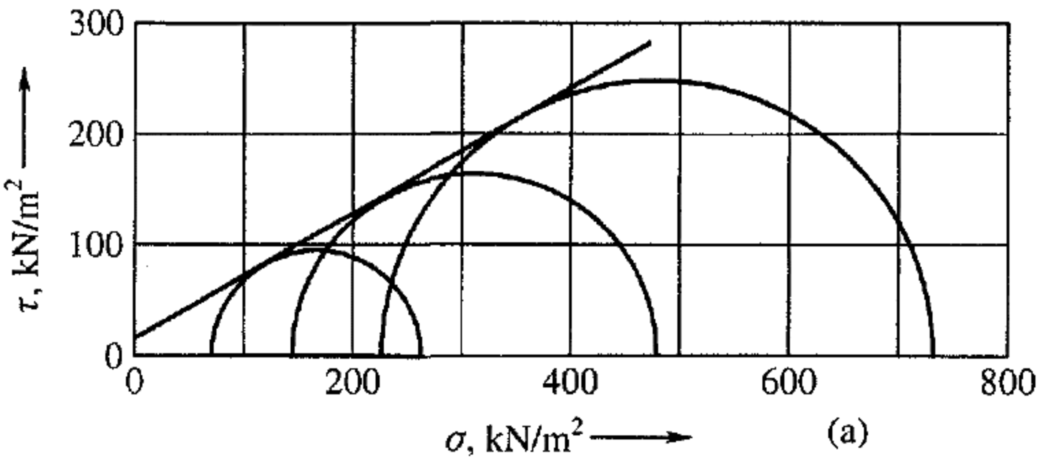
Solution

The values of the effective principal stresses $\sigma'_1$and σ'_3 at failure are tabulated below

Test No.	σ_3 (kN/m^2)	σ_1 (kN/m^2)	σ'_3 (kN/m^2)	σ'_1 (kN/m^2)
1	150	342	70	262
2	300	641	146	487
3	450	954	228	732

The Mohr circles regarding effective stresses and the failure envelope are drawn in Figure. The shear strength parameters as measured are:

c'=16 kN/m^2; ϕ'= 29°



Example (13)

The following results were obtained at failure in a series of triaxial tests on specimens of a saturated clay initially 38 mm in diameter and 76 mm long. Determine the values of the shear strength parameters with respect to (a) total stress, and (b) effective stress.

Test type	σ_3 (kN/m^2)	Axial load (N)	Axial compression (mm)	Volume change (cm^3)
Undrained	200	222	9.83	-
	400	215	10.06	-
	600	226	10.28	-
drained	200	467	10.81	6.6
	400	848	12.26	8.2
	600	1265	14.17	9.5

Solution

The principal stress difference at failure in each test is obtained by dividing the axial load by the cross-sectional area of the specimen at failure. The corrected cross-sectional area is calculated from Equation below

$A_o h_o = Ah$

where A_o, h_o = initial cross-sectional area and height of sample respectively.

A, h = cross-sectional area and height respectively at any stage of loading

If Δh is the compression of the sample, the strain is

$$\varepsilon = \frac{\Delta h}{h_0} \quad \text{since} \quad \Delta h = h_0 - h, \text{ we may write}$$

$$A_0 h_0 = A(h_0 - \Delta h)$$

$$\text{Therefore,} \quad A = \frac{A_0 h_0}{h_0 - \Delta h} = \frac{A_0}{1 - \Delta h / h_0} = \frac{A_0}{1 - \varepsilon}$$

The average vertical stress at any stage of loading may be written as

$$\sigma_1 = \frac{P}{A} = \frac{P(1-\varepsilon)}{A_0}$$

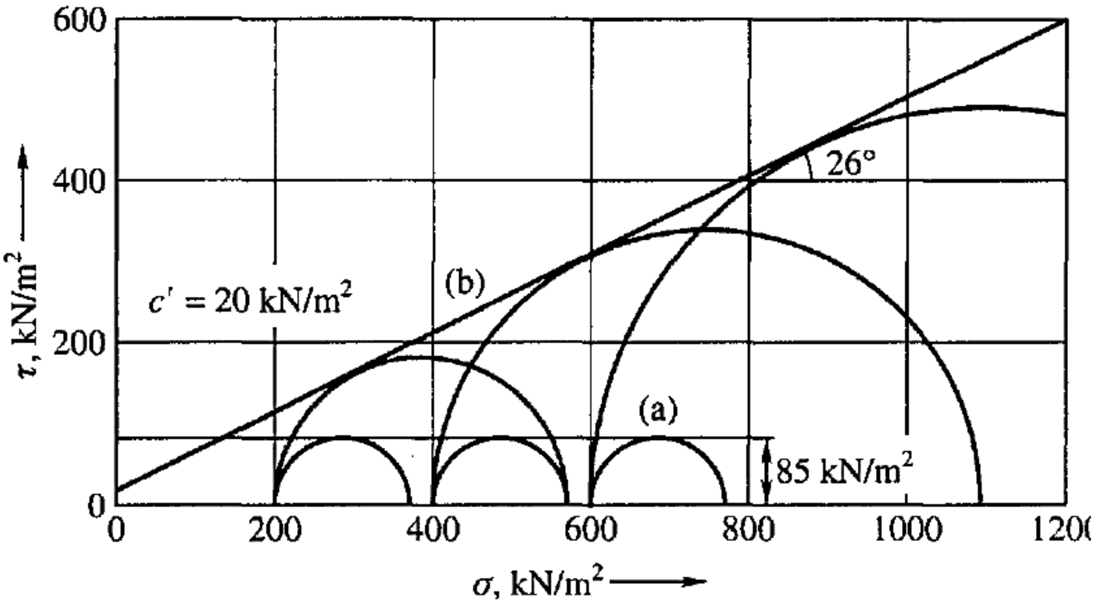
There is no volume change during an undrained test on a saturated clay. The initial values of length, area and volume for each specimen are h_o = 76 mm, A_o = 11.35 cm^2; V_O = 86.0 cm^3 respectively

The Mohr circles at failure and the corresponding failure envelopes for both series of tests are shown in Figure below. In both cases, the failure envelope is the line nearest to the common tangent to the Mohr circles. The total stress parameters representing the undrained strength of the clay are:

c_u = 85 kN/m^2; ϕ_u = 0

The effective stress parameters, representing the drained strength of the clay, are: c' = 20 kN/m^2; ϕ = 26°

Test type	σ_3 (kN/m^2)	$\Delta h/h_o$	$\Delta V/V_o$	Area (corrected) cm^2	σ_1 - σ_3 (kN/m^2)	σ_1 (kN/m^2)
Undrained	200	0.129		13.04	170	370
	400	0.132		13.09	160	564
	600	0.135		13.12	172	772
drained	200	0.142	0.077	12.22	382	582
	400	0.161	0.095	12.25	691	1091
	600	0.186	0.110	12.40	1020	1620



Example (14)

An embankment is being constructed of soil whose properties are c' = 50 kN/m^2, ϕ' = 21° (all effective stresses), and γ= 16 kN/m^3. The pore pressure parameters as determined from triaxial tests are A = 0.5, and B = 0.9. Find the shear strength of the soil at the base of the embankment just after the height of fill has been raised from 3m to 6m. Assume that the dissipation of pore pressure during this stage of construction is negligible and that the lateral pressure at any point is one-half of the vertical pressure.

Solution

The equation for pore pressure is

$$\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$

$\Delta\sigma_1$ = vertical pressure due to 3m of fill = 3 * 16 = 48 kN/m^2

$\Delta\sigma_3$ = 48/2 = 24 kN/m^2

$$\Delta u = 0.9[24 + 0.5(24)] = 32\ kN/m^2$$

σ_1 = 3 * 16 = 48 kN/m^2

$\sigma' = \sigma_1 + \Delta\sigma_1 - \Delta u$ = 48 +48 – 32 = 64 kN/m^2

$\tau_f = c' + \sigma' \tan\phi'$ = 50 + 64 * tan 21 = 74.57 kN/m^2

6.5.3 Unconsolidated-Undrained Triaxial Test

In unconsolidated-undrained tests, drainage from the soil specimen is not permitted during the application of chamber pressure σ_3. The test specimen is sheared to failure by the application of deviator stress, ($\Delta\sigma_d$), and drainage is prevented. Because drainage is not allowed at any stage, the test can be performed quickly. Because of the application of chamber confining pressure σ_3, the pore water pressure in the soil specimen will increase by u_c. A further increase in the pore water pressure (Δu_d) will occur because of the deviator stress application. Hence, the total pore water pressure u in the specimen at any stage of deviator stress application can be given as:

$$u = u_c + \Delta u_d$$

$$u_c = B\sigma_3 \text{ and } \Delta u_d = \overline{A}\Delta\sigma_d, \text{ so}$$

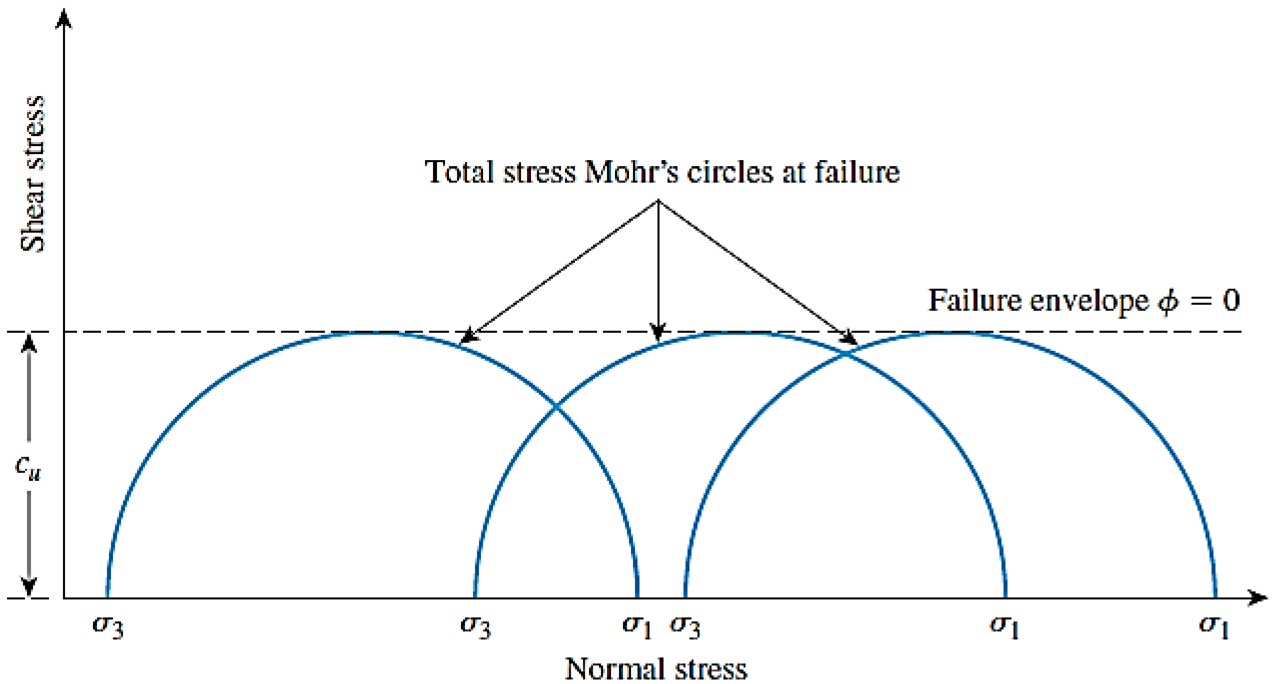
$$u = B\sigma_3 + \overline{A}\Delta\sigma_d = B\sigma_3 + \overline{A}(\sigma_1 - \sigma_3)$$

This test usually is conducted on clay specimens and depends on a very important strength concept for cohesive soils if the soil is fully saturated. The failure envelope for the total stress Mohr's circles is shown in Figure below becomes a horizontal line and hence is called a $\phi = 0$ condition.

$\tau_f = c = c_u$

Where: c_u is the undrained shear strength and is equal to the radius of the Mohr's circles.

Note that the $\phi = 0$ concept applies to only saturated clays and silts.



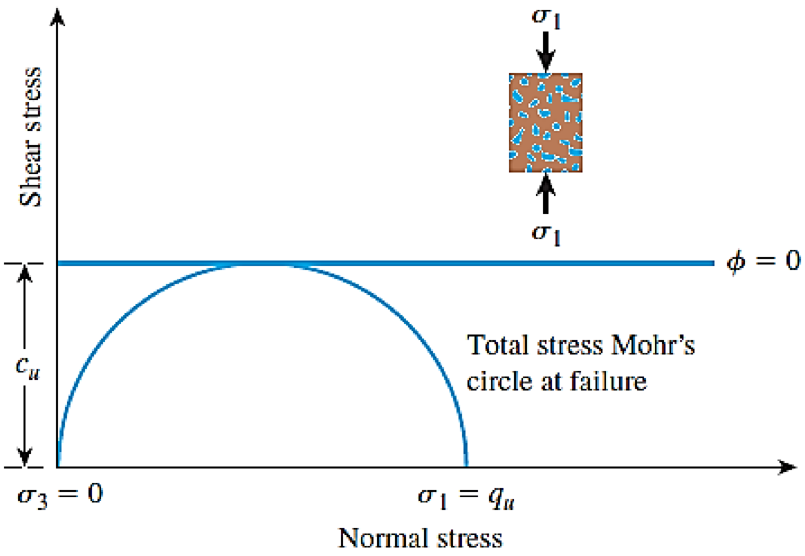
6.6 Unconfined Compression Test on Saturated Clay

The unconfined compression test is a special type of unconsolidated-undrained test that is commonly used for clay specimens. In this test, the confining pressure σ_3 is 0. An axial load is rapidly applied to the specimen to cause failure. At failure, the total minor principal stress is zero, and the total major principal stress is σ_1.

$$\tau_f = \frac{\sigma_1}{2} = \frac{q_u}{2} = c_u$$

Where: q_u is the unconfined compression strength.

The table gives the approximate consistencies of clays on the basis of their unconfined compression strength. A photograph of unconfined compression test equipment is shown in Figure



General Relationship of Consistency and Unconfined Compression Strength of Clays

Consistency	q_u kN/m²
Very soft	0–25
Soft	25–50
Medium	50–100
Stiff	100–200
Very stiff	200–400
Hard	>400

Example (15)

Boreholes reveal that a thin layer of alluvial silt exists at a depth of 15.25m below the surface of the ground. The soil above this level has an average dry unit weight of 15 kN/m^3 and an average water content of 30%. The water table is approximately at the surface. Tests on undisturbed samples give the following data: c_u = 48 kN/m^2, ϕ_u = 13°, c_d = 41.25 kN/m^2, ϕ_d = 23°. Estimate the shearing resistance of the silt on a horizontal plane (a) when the shear stress builds up rapidly, and (b) when the shear stress builds up very slowly.

Solution

Total unit weight = $\gamma_t = \gamma_d * (1 + \omega)$ = 15 * 1.3 = 19.5 kN/m^3

Submerged unit weight = γ_b = 19.5 - 9.81 = 9.69 kN/m^3

Total normal pressure at 15.25m depth = 15.25*19.5 = 297.38 kN/m^2

Effective pressure at 15.25m depth = 15.25 * 9.69 = 147.78 kN/m^2

(a) For rapid biuld – up, use the properties of undrained state and total pressure. At totoal pressure of 297.38 kN/m^2

Shear strength $\tau_f = c + \sigma \tan\phi$ = 48 + 297 tan 13 = 116.57 kN/m^2

(b) For slow build – up use effective stress properties

At an effective stress of 147.78 kN/m^2

Shear strength = 41.25 + 147.78 tan 23 = 103.98 kN/m^2

Example (16)

When an undrained triaxial compression test was conducted on specimens of clayey silt, the following results were obtained:

Speciemen No.	1	2	3
σ_3 (kN/m^2)	17	44	56
σ_1 (kN/m^2)	157	204	255
u (kN/m^2)	12	20	22

Determine the values of shear parameters considering (a) total stresses and (b) effective stresses.

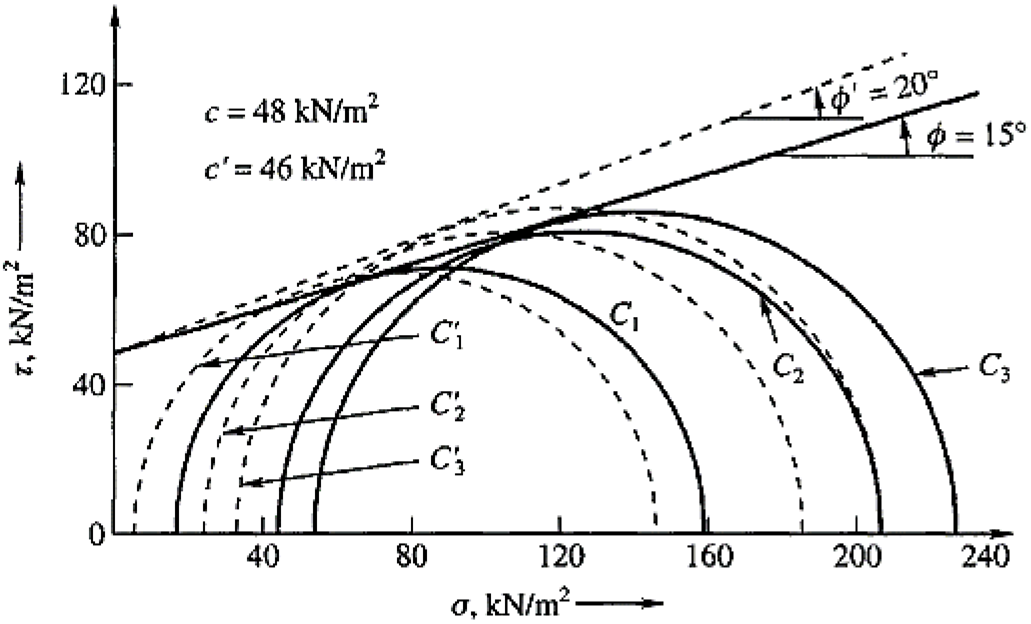
Solution

(a) Total stresses

For a solution with total stresses, draw Mohr circles C1, C2 and C3 for each of the specimens using the corresponding principal stresses σ_1 and σ_3.

Draw a Mohr envelope tangent to these circles as shown in Figure. Now from the figure

c = 48 kN/m^2, ϕ= 15°



(b) With effective stresses

The effective principal stresses may be found by subtracting the pore pressures u from the total principal stresses as given below.

Speciemen No.	1	2	3
σ_3 (kN/m^2)	5	24	34
σ_1 (kN/m^2)	145	184	204

As before draw Mohr circles C'1 C′2 and C′3 for each of the specimens as shown in Figure. Now from the figure

c' = 46 kN/m^2, ϕ'= 20°

Example (17)

The soil has an unconfined compressive strength of 120 kN/m^2. In a triaxial compression test, a specimen of the same soil when subjected to a chamber pressure of 40 kN/m^2 failed at an additional stress of 160 kN/m^2. Determine:

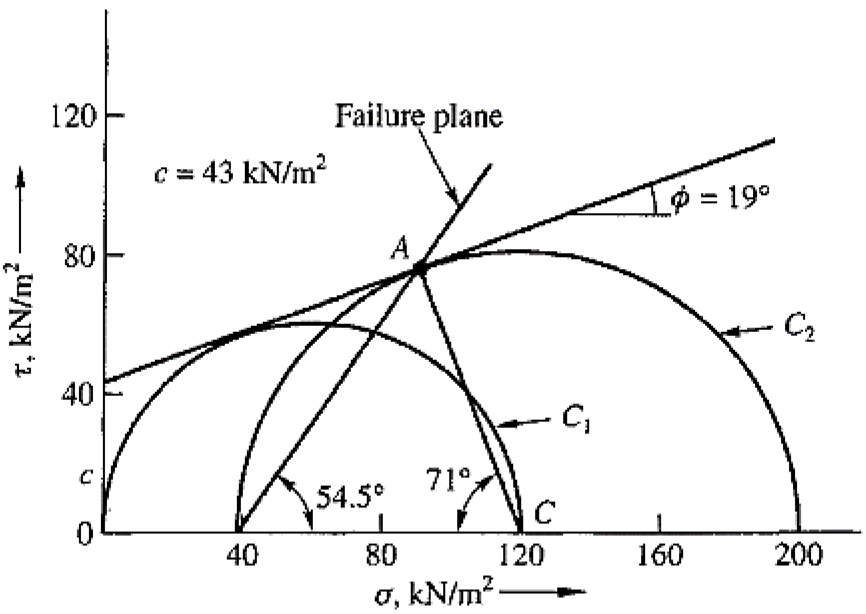
(i) The shear strength parameters of the soil, (ii) the angle made by the failure plane with the axial stress in the triaxial test.

Solution

There is one unconfined compression test result and one triaxial compression test result. Hence two Mohr circles, C1 and C2 may be drawn as shown in Figure. For Mohr circle C1, $\sigma_3 = 0$ and $\sigma_1 = 120$ kN/m^2, and for Mohr circle C2, $\sigma_3 = 40$ kN/m^2 and $\sigma_1 = (40 + 160) = 200$ kN/m^2. A common tangent to these two circles is the Mohr envelope which gives

(i) c = 43 kN/m^2 and $\phi = 19°$

(ii) For the triaxial test specimen, A is the point of tangency for Mohr circle C2, and C is the center of circle C2. The angle made by AC with the abscissa is equal to twice the angle between the failure plane and the axis of the sample, 2α. From Figure, $2\alpha = 26 = 71°$ and $\alpha = 35.5°$. The angle made by the failure plane with the σ -axis is $\theta = 90° - 35.5° = 54.5°$.



Example (18)

A cylindrical sample of saturated clay 4 cm in diameter and 8 cm high was tested in an unconfined compression apparatus. Find the unconfined compression strength, if the specimen failed at an axial load of 360 N when the axial deformation was 8 mm. Find the shear strength parameters if the angle made by the failure plane with the horizontal plane was recorded as 50°.

Solution

The unconfined compression strength of the soil is given by

$$\sigma_1 = \frac{P(1-\varepsilon)}{A_o}, \quad \text{where } P = 360\ N$$

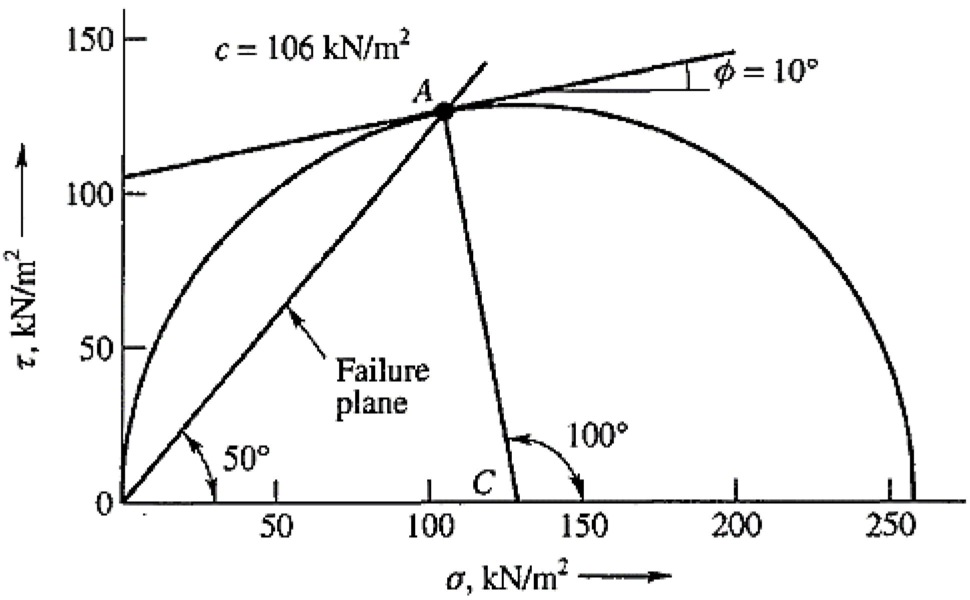
$$A_o = \frac{3.14}{4} \times (4)^2 = 12.56\ \text{cm}^2, \quad \varepsilon = \frac{0.8}{8} = 0.1$$

$$\text{Therefore } \sigma_1 = \frac{360(1-0.1)}{12.56} = 25.8\ \text{N/cm}^2 = 258\ \text{kN/m}^2$$

Now $\phi = 2\theta - 90°$, where $\theta = 50°$. Therefore $\phi = (2 \times 500 - 90° = 10°$.

Draw the Mohr circle as shown in Figure. ($\sigma_3 = 0$ and $\sigma_1 = 258$ kN/m^2) and from the center C of the circle, draw CA at $2\theta = 100°$. At point A, draw a tangent to the circle. The tangent is the Mohr envelope which gives

c = 106 kN/m^2, and ϕ=10°



Example (19)

A cylindrical sample of soil having a cohesion of 80 kN/m^2 and an angle of internal friction of 20° is subjected to a cell pressure of 100 kN/m^2.

Determine: (i) the maximum deviator stress (σ_1 - σ_3) at which the sample will fail, and (ii) the angle made by the failure plane with the axis of the sample.

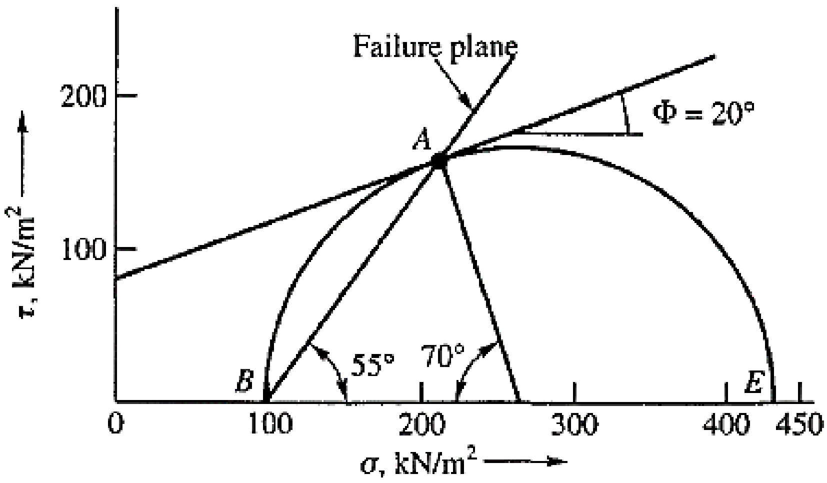
Solution

Graphically

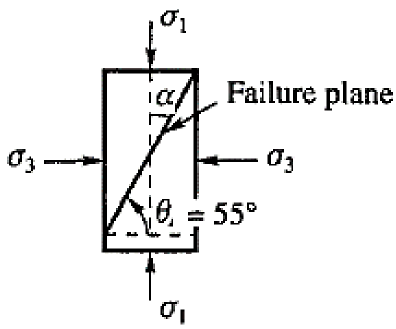
σ_3 = 100 kN/m^2, ϕ = 20°, and c = 80 kN/m^2.

A Mohr circle and the Mohr envelope can be drawn as shown in Figure. The circle cuts the σ-axis at B (= σ_3), and at E (=σ_1). Now σ_1 = 433 kN/m^2, and σ_3 = 100 kN/m^2.

(σ_1, - σ_3) = 433 - 100 = 333 kN/m^2.



(a)



(b)

Analytical

$$\sigma_1 = \sigma_3 \tan^2\left(45° + \frac{\phi}{2}\right) + 2c \tan\left(45° + \frac{\phi}{2}\right)$$

Substituting the known values, we have

$\tan(45° + \phi/2) = \tan(45° + 10) = \tan 55° = 1.428$

$\tan^2(45° + \phi/2) = 2.04.$

Therefore,

$\sigma_1 = 100 \times 2.04 + 2 \times 80 \times 1.428 \approx 433\ \text{kN/m}^2$

$(\sigma_1 - \sigma_3) = (433 - 100) = 333\ \text{kN/m}^2$

If α = angle made by failure planes with the axis of the sample.

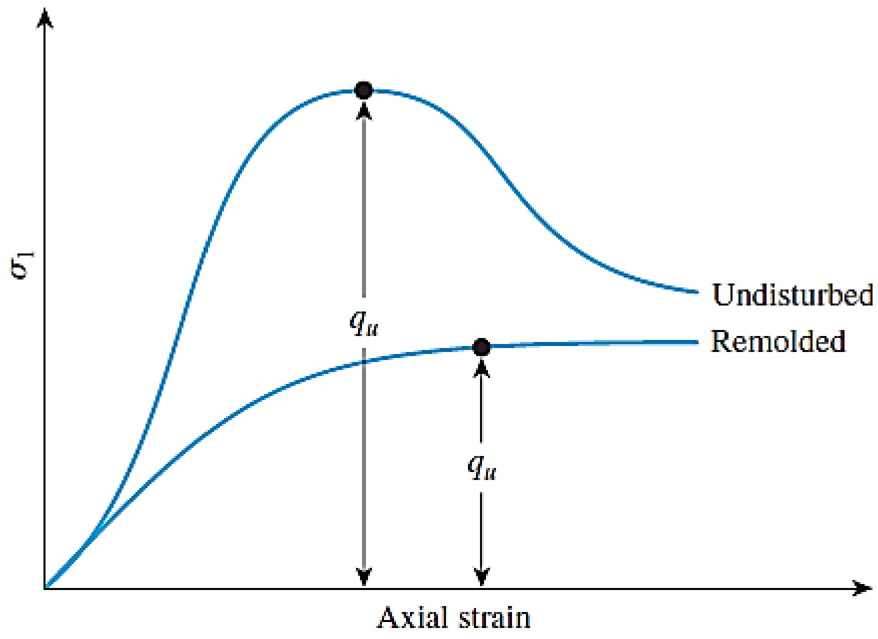
$2\alpha = 90 - \phi = 90 - 20 = 70°$ or $\alpha = 35°$.

Therefore, the angle made by the failure plane with the σ-axis is

$\theta = 90 - 35 = 55°$

6.7 Sensitivity of Clay

For many naturally deposited clay soils, the unconfined compression strength is reduced greatly when the soils are tested after remolding without any change in the moisture content, as shown in Figure. This property of clay soils is called sensitivity. The degree of sensitivity may be defined as the ratio of the unconfined compression strength in an undisturbed state to that in a remolded state, or



$$S_t = \frac{c_u\ (\text{undisturbed})}{c_u\ (\text{remolded})} = \frac{\tau_f\ (\text{undisturbed})}{\tau_f\ (\text{remolded})}$$

The sensitivity ratio of most clays ranges from about 1 to 8; however, highly flocculent marine clay deposits may have sensitivity ratios ranging from about 10 to 80. Some clays turn to viscous fluids upon remolding. These clays are found mostly in the previously glaciated areas of North America and

Scandinavia. Such clays are referred to as quick clays. Rosenqvist (1953) classified clays on the basis of their sensitivity as follows:

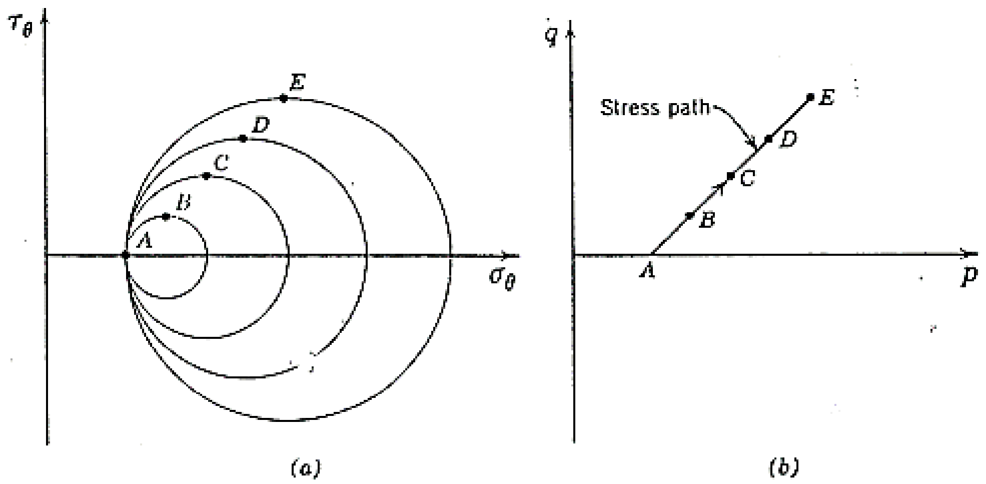
Sensitivity	Classification
1	Insensitive
1–2	Slightly sensitive
2–4	Medium sensitive
4–8	Very sensitive
8–16	Slightly quick
16–32	Medium quick
32–64	Very quick
>64	Extra quick

6.8 Stress Path

When we have many soil samples, it is difficult to study the change of stresses in the soil. Thus, the results of triaxial tests can be represented by diagrams called **stress paths**. In this diagram, the results for one Mohr's circle (state of stress) will transform to a point.

$$p' = \frac{\sigma_1' + \sigma_3'}{2} \quad , \quad q' = \frac{\sigma_1' - \sigma_3'}{2}$$

A stress path is a line or curve that connects a series of points, each of which represents a successive stress state experienced by a soil specimen during the progress of a test.



In geostatic stresses, the results can be expressed as:

$$p' = \frac{\sigma'_v + \sigma'_h}{2} \qquad q' = \pm\frac{\sigma'_v - \sigma'_h}{2}$$

Example (20)

If the initial stresses is: $\sigma_v = \sigma_h$, find the stress path for the following cases.

(a) $\Delta\sigma_v > 0$, $\Delta\sigma_h = 0$ (b) $\Delta\sigma_v = 0$, $\Delta\sigma_h > 0$ (c) $\Delta\sigma_h = -\Delta\sigma_v$ (d) $\Delta\sigma_v = \Delta\sigma_h$ (e) $\Delta\sigma_h = 0.25\ \Delta\sigma_v$

Solution

Initial stresses

$$p = \frac{\sigma_v + \sigma_h}{2} = \frac{\sigma_v + \sigma_v}{2} = \sigma_v, \quad q = \frac{\sigma_v - \sigma_h}{2} = \frac{\sigma_v - \sigma_v}{2} = 0$$

(a) $\Delta\sigma_v > 0$, $\Delta\sigma_h = 0$

$$\Delta p = \frac{\Delta\sigma_v + \Delta\sigma_h}{2} = \frac{\Delta\sigma_v + 0}{2} = \frac{\Delta\sigma_v}{2}, \qquad \Delta q = \frac{\Delta\sigma_v - \Delta\sigma_h}{2} = \frac{\Delta\sigma_v - 0}{2} = \frac{\Delta\sigma_v}{2}$$

$$slope = \frac{\Delta q}{\Delta p} = \frac{\frac{\Delta\sigma_v}{2}}{\frac{\Delta\sigma_v}{2}} = 1$$

(b) $\Delta\sigma_v = 0$, $\Delta\sigma_h > 0$

$$\Delta p = \frac{\Delta\sigma_v + \Delta\sigma_h}{2} = \frac{0 + \Delta\sigma_h}{2} = \frac{\Delta\sigma_h}{2}, \qquad \Delta q = \frac{\Delta\sigma_v - \Delta\sigma_h}{2} = \frac{0 - \Delta\sigma_h}{2} = -\frac{\Delta\sigma_h}{2}$$

$$slope = \frac{\Delta q}{\Delta p} = \frac{\frac{\Delta\sigma_h}{2}}{-\frac{\Delta\sigma_h}{2}} = -1$$

(c) $\Delta\sigma_h = -\Delta\sigma_v$

$$\Delta p = \frac{\Delta\sigma_v + \Delta\sigma_h}{2} = \frac{\Delta\sigma_v + (-)\Delta\sigma_v}{2} = 0, \Delta q = \frac{\Delta\sigma_v - \Delta\sigma_h}{2} = \frac{\Delta\sigma_v - (-)\Delta\sigma_v}{2} = \Delta\sigma_v$$

$$slope = \frac{\Delta q}{\Delta p} = \frac{\Delta\sigma_v}{0} = \infty$$

(d) $\Delta\sigma_v = \Delta\sigma_h$

$$\Delta p = \frac{\Delta\sigma_v + \Delta\sigma_h}{2} = \frac{\Delta\sigma_v + \Delta\sigma_v}{2} = \Delta\sigma_v, \qquad \Delta q = \frac{\Delta\sigma_v - \Delta\sigma_h}{2} = \frac{\Delta\sigma_v - \Delta\sigma_v}{2} = 0$$

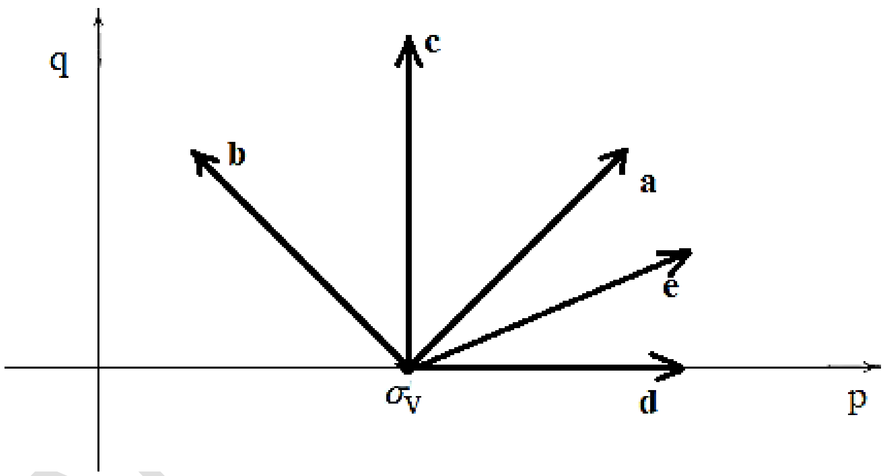
$$slope = \frac{\Delta q}{\Delta p} = \frac{0}{\Delta\sigma_v} = 0$$

(e) $\Delta\sigma_h = 0.25\,\Delta\sigma_v$

$$\Delta p = \frac{\Delta\sigma_v + \Delta\sigma_h}{2} = \frac{\Delta\sigma_v + 0.25\Delta\sigma_v}{2} = 0.625\Delta\sigma_v$$

$$\Delta q = \frac{\Delta\sigma_v - \Delta\sigma_h}{2} = \frac{\Delta\sigma_v - 0.25\Delta\sigma_v}{2} = 0.375\Delta\sigma_v$$

$$slope = \frac{\Delta q}{\Delta p} = \frac{0.375\Delta\sigma_v}{0.625\Delta\sigma_v} = \frac{3}{5}$$



Example (21)

If the initial stresses is $\sigma_v = \sigma_h = 0$ (at soil surface), find the stress path for the condition for (k_o).

Solution

$\sigma_h = k_o\,\sigma_v$

Initially, p = 0, q = 0

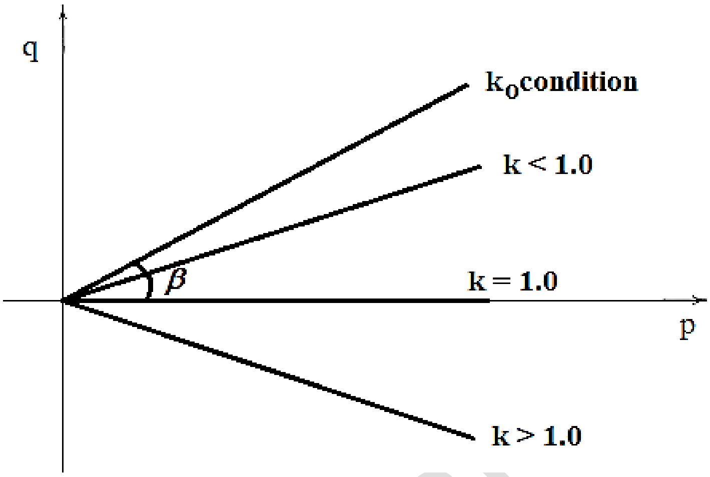
$$\Delta p = \frac{\sigma_v + k_o\sigma_v}{2} = \frac{(1+k_o)}{2}\sigma_v, \qquad \Delta q = \frac{\sigma_v - k_o\sigma_v}{2} = \frac{(1-k_o)}{2}\sigma_v$$

$$slope = \frac{\Delta q}{\Delta p} = \frac{(1-k_o)}{(1+k_o)}$$

k_o may be < 0, or > 0, or = 0 as shown

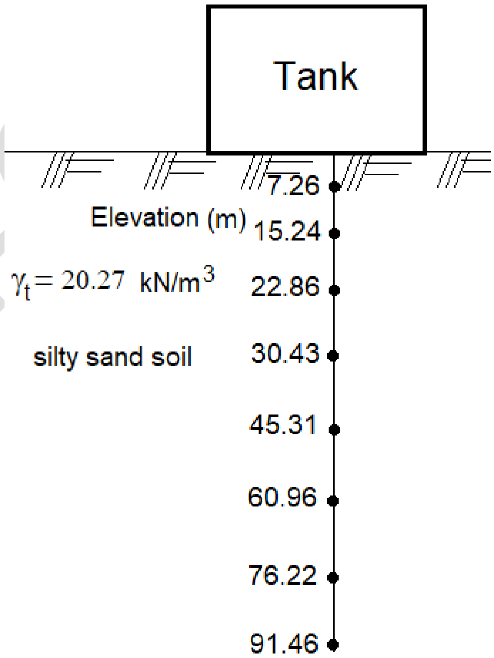
if q/p = tanβ, then

$$k_o = \frac{1 - tan\beta}{1 + tan\beta}$$



Example (22)

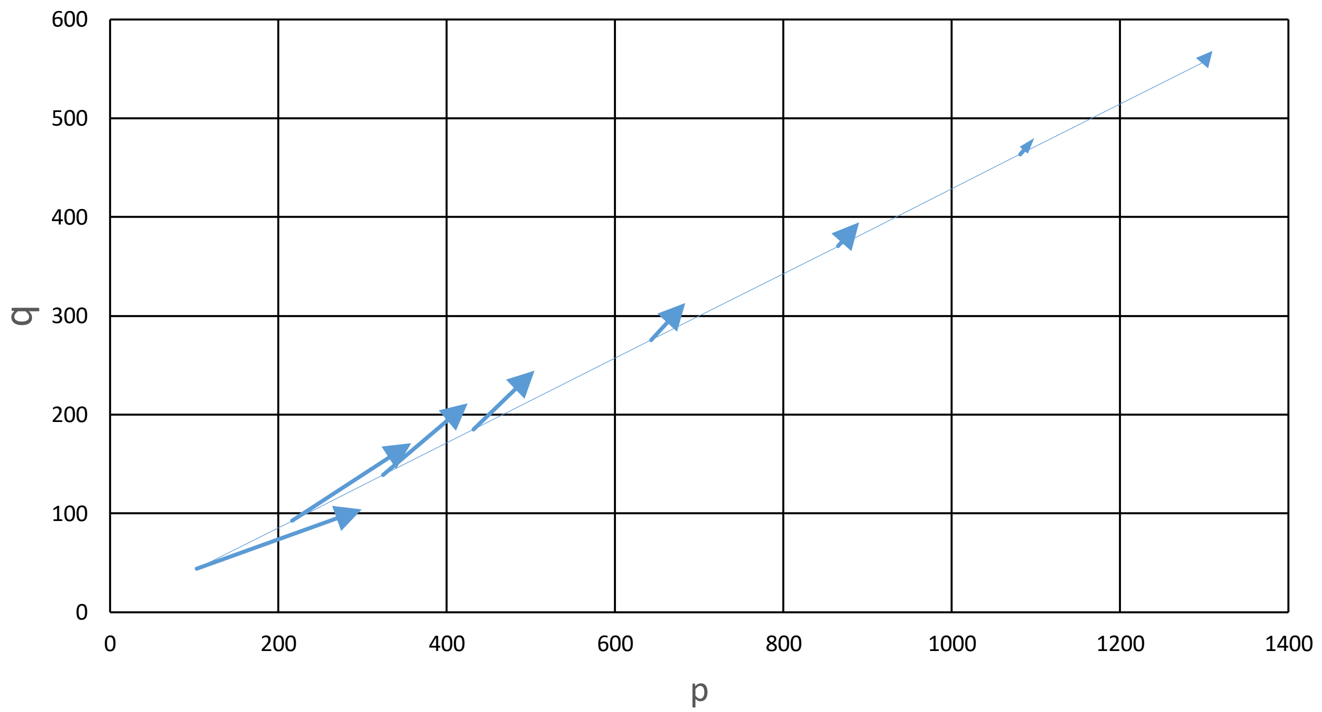
For the soil profile shown and the loading condition draw the stress path for points A to H, if $k_o = 0.4$.



Solution

- Find the initial vertical stress, $\sigma_v = \gamma h$
- Find the initial horizontal stress, $\sigma_h = k_o \sigma_v$
- Find p_i, q_i
- Add vertical and horizontal stresses (given) to initial stresses.
- Find p_f, q_f

point	Elev. (m)	σ_v kN/m²	σ_h kN/m²	p_i kN/m²	q_i kN/m²	**$\Delta\sigma_v$ kN/m²**	**$\Delta\sigma_h$ kN/m²**	σ_{vf} kN/m²	σ_{hf} kN/m²	p_f kN/m²	q_f kN/m²
A	7.26	147.16	58.86	103.01	44.15	**256**	**136**	403.16	194.86	299.01	104.15
B	15.24	308.91	123.57	216.24	92.67	**220**	**63**	528.91	186.57	357.74	171.17
C	22.86	463.37	185.35	324.36	139.01	**173**	**28**	636.37	213.35	424.86	211.51
D	30.43	616.82	246.73	431.77	185.04	**132**	**13**	748.82	259.73	504.27	244.54
E	45.31	918.43	367.37	642.90	275.53	**78**	**3**	996.43	370.37	683.40	313.03
F	60.96	1235.66	494.26	864.96	370.70	**49**	**1**	1284.66	495.26	889.96	394.70
G	76.22	1544.98	617.99	1081.49	463.49	**33**	**0**	1577.98	617.99	1097.99	479.99
H	91.46	1853.89	741.56	1297.73	556.17	**24**	**0**	1877.89	741.56	1309.73	568.17

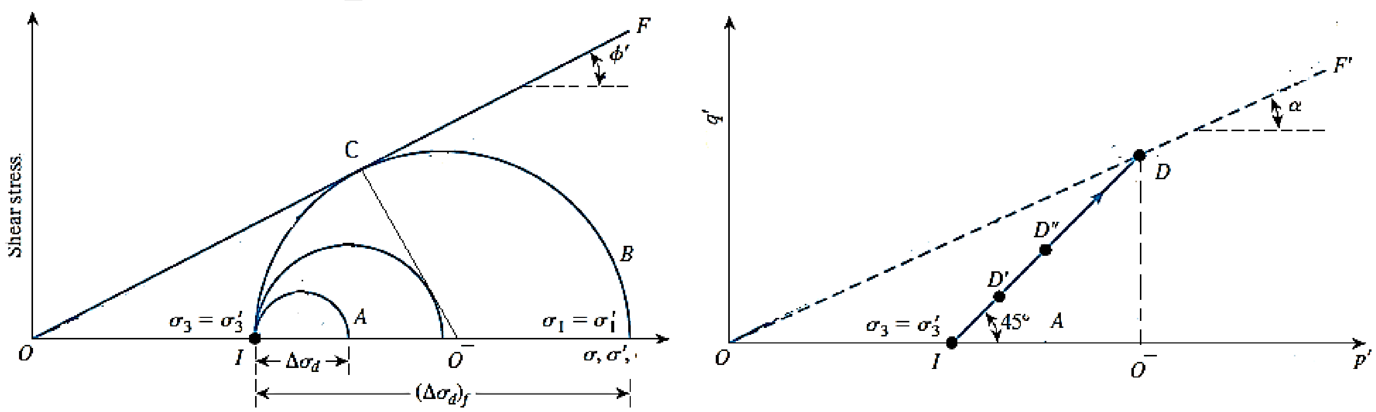


What is the tangent of line connected initial p and q?

6.9 Stress Path for Triaxial Result Tests

This type of stress path plot can be explained with the aid of Figure. Let us consider a normally consolidated clay specimen subjected to the anisotropically consolidated drained triaxial test. At the beginning of the application of deviator stress, $\sigma'_1 = \sigma'_3, = \sigma_3$, so

$$p' = \frac{\sigma'_3 + \sigma'_3}{2} = \sigma'_3 = \sigma_3 \qquad q' = \frac{\sigma'_3 - \sigma'_3}{2} = 0$$



For this condition, p′ and q′ will plot as a point (that is, I in Figure). After deviator stress application, for Mohr's circle A in Figure. The values of p′ and q′ for this stress condition are:

$$p' = \frac{\sigma_1' + \sigma_3'}{2} = \frac{(\sigma_3' + \Delta\sigma_d) + \sigma_3'}{2} = \sigma_3' + \frac{\Delta\sigma_d}{2} = \sigma_3 + \frac{\Delta\sigma_d}{2}$$

$$q' = \frac{(\sigma_3' + \Delta\sigma_d) - \sigma_3'}{2} = \frac{\Delta\sigma_d}{2}$$

If these values of p′ and q′ were plotted in Figure, they would be represented by point D′ at the top of the Mohr's circle. So, if the values of p′ and q′ at various stages of the deviator stress application are plotted, and these points are joined, a straight line like ID will result. The straight line ID is referred to as the stress path. Note that the line ID makes an angle of 45° with the horizontal. Point D represents the failure condition of the soil specimen in the test. Also, we can see that Mohr's Circle B represents the failure stress condition.

For normally consolidated clays, the failure envelope given by $\tau_f = \sigma' \tan\phi'$.

This is the line OF in Figure. A modified failure envelope now can be defined by line OF′. The equation of the line can be expressed as:

$$q' = p' \tan\alpha$$

where α = the angle that the modified failure envelope makes with the horizontal

$$\frac{DO'}{OO'} = \tan\alpha \qquad \tan\alpha = \frac{\dfrac{\sigma_1' - \sigma_3'}{2}}{\dfrac{\sigma_1' + \sigma_3'}{2}} = \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'}$$

$$\frac{CO'}{OO'} = \sin\phi'$$

$$\sin\phi' = \frac{\dfrac{\sigma_1' - \sigma_3'}{2}}{\dfrac{\sigma_1' + \sigma_3'}{2}} = \frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'}$$

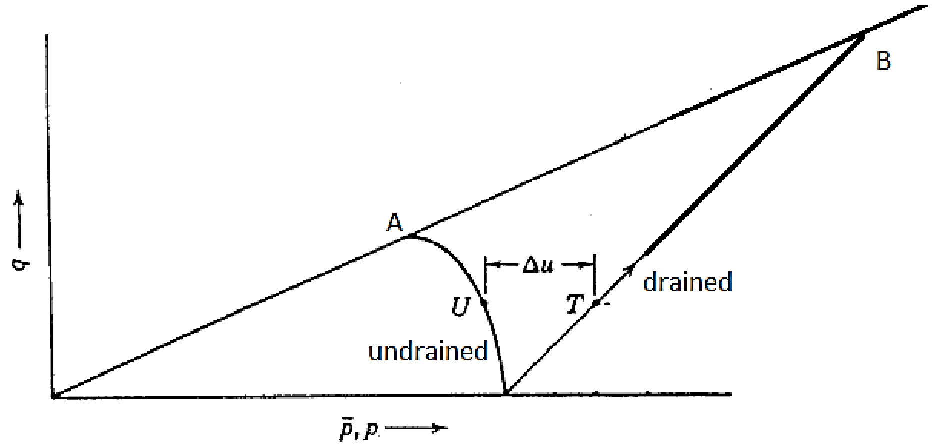
$$\sin\phi' = \tan\alpha \qquad \phi' = \sin^{-1}(\tan\alpha)$$

For consolidated undrained tests

Initialy (isotropic)

$p' = \sigma'_3$, $q = q' = 0$

After shearing



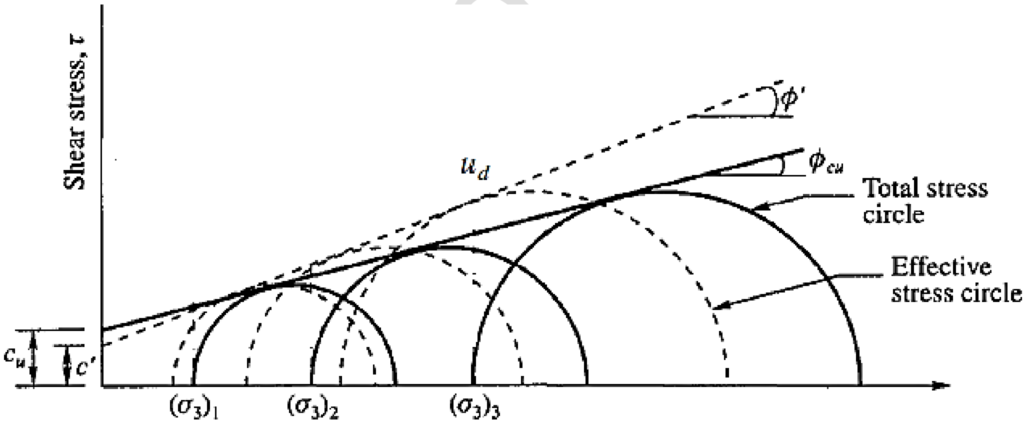
$$\sigma'_1 = \sigma_3 + \Delta\sigma_d - \Delta u_d$$

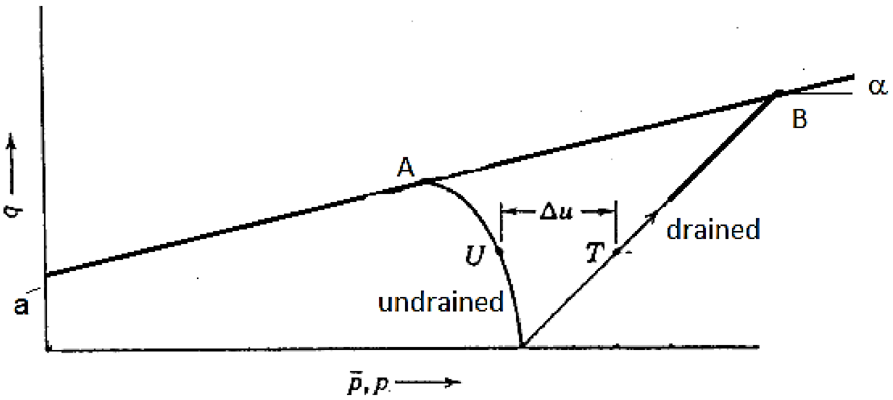
$$\sigma'_3 = \sigma_3 - \Delta u_d$$

$$p' = \frac{\sigma'_1 + \sigma'_3}{2} = \sigma_3 + \frac{\Delta\sigma_d}{2} - \Delta \qquad p' = \sigma_3 + \frac{(\Delta\sigma_d)_f}{2} - (\Delta u_d)_f$$

$$q' = \frac{\sigma'_1 - \sigma'_3}{2} = \frac{\Delta\sigma_d}{2} \qquad q' = \frac{(\Delta\sigma_d)_f}{2}$$

For overconsolidated clay





$$c' cot\phi' = \frac{\frac{\sigma_1 - \sigma_3}{2}}{sin\phi'} - p'$$

$$\frac{\sigma_1 - \sigma_3}{2} = q$$

$$c' = \frac{1}{cot\phi'}\left(\frac{q}{sin\phi'} - p'\right)$$

$$c' = \frac{q}{cos\phi'} - \frac{p' sin\phi'}{cos\phi'}$$

At p' = 0, q = a

$$a' = c' cos\phi'$$

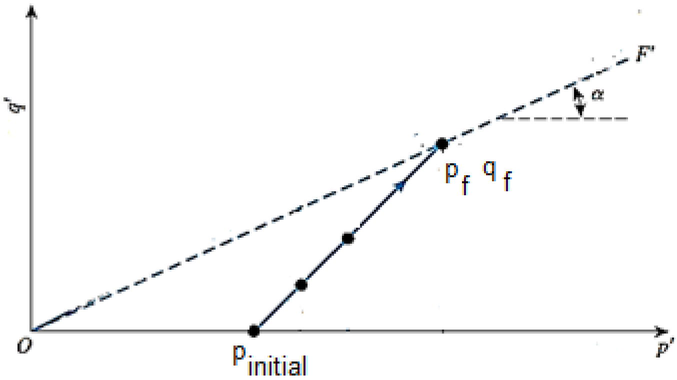
Example (23)

A specimen of normally onsolidated clay is consolidated to 689 kN/m^2 and then failed by decresing σ'_3 while σ'_1 held constant. Find q_f and p'_f, knowning that ϕ' = 22°

Solution

$\sin\phi' = \tan\alpha' \rightarrow \sin 22° = \tan\alpha' \rightarrow \alpha = 20.5°$

initial q = 0, p′ = 689 kN/m^2



$$\frac{q-0}{p-0} = \tan 20.5$$

q = 0.375 p′ (1)

$$\Delta q = \frac{\Delta\sigma_1 - \Delta\sigma_3}{2} = \frac{-(-\Delta\sigma_3)}{2} = \frac{\Delta\sigma_3}{2}$$

$$\Delta p = \frac{\Delta\sigma_1 + \Delta\sigma_3}{2} = \frac{(-\Delta\sigma_3)}{2} = \frac{-\Delta\sigma_3}{2}$$

$$\frac{\Delta q}{\Delta p} = -1 \rightarrow \frac{q-0}{p-689} = -1 \rightarrow q = 689 - p \qquad (2)$$

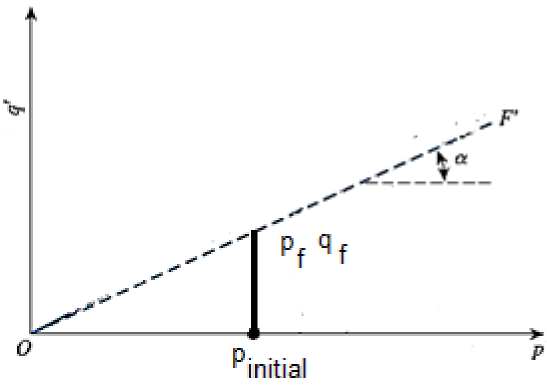
Solve the Eq. 1 and 2

p'_f = 501.1 kN/m^2, q = 188 kN/m^2

Example (24)

A specimen of normally onsolidated clay is consolidated to 414 kN/m^2 and then failed by decresing σ'_3 and increasing σ'_1 in such away that p′ remain constant Find q_f and p'_f, knowning that $\phi' = 22°$

Solution



$\sin\phi' = \tan\alpha' \rightarrow \sin 22° = \tan\alpha' \rightarrow \alpha = 20.5°$

initial q = 0, p′ = 414 kN/m^2

$p_f = 414$

$$\frac{q_f}{p_f} = \tan 20.5$$

$q_f = 414 * \tan 20.5 = 154.78$ kN/m^2

Homework Chapter 6

(1) Following data are given for a direct shear test conducted on dry sand:

Specimen dimensions: 63 mm * 63 mm * 25 mm (height)

Normal stress: 105 kN/m^2

Shear force at failure: 300 N

a. Determine the angle of friction, ϕ'

b. For a normal stress of 180 kN/m^2, what shear force is required to cause failure?

Ans. ϕ' = 35.5, shear force (S) = 509.5 N

(2) Consider the specimen in Problem (1b).

a. What are the principal stresses at failure?

b. What is the inclination of the major principal plane with the horizontal?

Ans. (a) σ_3 = 115 kN/m^2, σ_1 = 420 kN/m^2 (b) α = 65°

(3) For a dry sand specimen in a direct shear test box, the following are given:

Size of specimen: 63.5 mm * 63.5 mm * 31.75 mm (height)

Angle of friction: 33°

Normal stress: 193 kN/m^2

Determine the shear force required to cause failure. **Ans. S = 0.505 kN**

(4) The following are the results of four drained direct shear tests on undisturbed normally consolidated clay samples having a diameter of 50 mm and height of 25 mm.

Test No.	Normal force (N)	Shear force at failure (N)
1	67	23.3
2	133	46.6
3	213	73.5
4	369	132.3

Draw a graph for shear stress at failure against the normal stress and determine the drained angle of friction from the graph.

Ans. From graph ϕ = 19.5

(5) Repeat Problem (4) with the following data. Given: Specimen diameter = 50 mm; specimen height = 25 mm. **Ans. From graph ϕ = 29**

Test No.	Normal force (N)	Shear force at failure (N)
1	250	139
2	375	209
3	450	250
4	540	300

(6) Consider the clay soil in Problem 5. If a drained triaxial test is conducted on the same soil with a chamber confining pressure of 208 kN/m^2, what would be the deviator stress at failure? **Ans. $\Delta\sigma_{d(f)}$ = 392 kN/m^2**

(7) For the triaxial test on the clay specimen in Problem 6

a) What is the inclination of the failure plane with the major principal plane?

b) Determine the normal and shear stress on a plane inclined at 30° with the major principal plane at failure. Also, explain why the specimen did not fail along this plane. **Ans. θ = 59.5°, τ_f = 169.7 kN/m^2, σ' = 502 kN/m^2**

(8) The relationship between the relative density, D_r, and the angle of friction, ϕ' of sand can be given as $\phi' = 28 + 0.18D_r$ (Dr in %). A drained triaxial test was conducted on the same sand with a chamber confining pressure of 150 kN/m^2. The sand sample was prepared at a relative density of 68%. Calculate the major principal stress at failure. **Ans. σ'_1 = 697.4 kN/m^2**

(9) For a normally consolidated clay specimen, the results of a drained triaxial test are as follows:

- Chamber-confining pressure =125 kN/m^2
- Deviator stress at failure = 175 kN/m^2

Determine the soil friction angle, ϕ'. **Ans. ϕ' = 24.3°**

(10) In a consolidated drained triaxial test on clay, the specimen failed at a deviator stress of 124 kN/m^2. If the effective stress friction angle is known to be 31°, what was the effective confining pressure at failure?

Ans. σ'_3 = 58.54 kN/m^2

(11) Consider the clay sample in Problem 10. A consolidated-undrained triaxial test was conducted on the same clay with a chamber pressure of 103 kN/m2. The pore pressure at failure $(\Delta u_d)_f$ = 33 kN/m2. What would be the major principal stress, σ'_1, at failure? **Ans. σ'_1 = 219.67 kN/m^2**

(12) Following are the results of consolidated-undrained triaxial tests on undisturbed soils retrieved from a 4-m-thick saturated clay layer in the field (γ_{sat} = 19 kN/m^3). a. Estimate graphically the Mohr–Coulomb shear strength parameters c′ and ϕ'. b. Estimate the shear strength in the middle of the clay layer. **Ans. (a) *c′* ≈ 40 kN/m2 and ϕ' ≈ 20° (b) τ = 46.7 kN/m^2**

Test No.	Champer pressure, σ_3 (kN/m^2)	Deviator stress, $(\Delta\sigma_d)_f$ (kN/m^2)	Pore pressure at failure $(\Delta u_d)_f$ (kN/m^2)
1	100	170	-15
2	200	260	-40
3	300	360	-80

(13) A consolidated-drained triaxial test was conducted on a normally consolidated clay with a chamber pressure, σ_3 = 172 kN/m^2. The deviator stress at failure, $(\Delta\sigma_d)_f$ = 227 kN/m^2. Determine:

a. The angle of friction, ϕ'

b. The angle u that the failure plane makes with the major principal plane

c. The normal stress, σ'_f, and the shear stress, τ_f, on the failure plane

Ans. (a) ϕ' ≈ 23.4° (b) θ = 56.7° (c) σ'_f = 240.9 kN/m^2, τ_f = 104.25 kN/m^2

(14) The results of two consolidated drained triaxial tests on clay are given below:

specimen	Chamber pressure, σ_3 (kN/m^2)	Deviator stress, $(\Delta\sigma_d)_f$ (kN/m^2)
1	105	220
2	210	400

Calculate the shear strength parameters. **Ans. ϕ' = 27.48° c′ = 12.48 kN/m^2**

(15) Consider the triaxial tests in Problem 14. (a) What are the normal and shear stresses on a plane inclined at 40° to the major principal plane for Specimen 1, (b) What are the normal and shear stresses on the failure plane at failure for Specimen 2.

Ans. (a) $\sigma' = 234.1$ kN/m^2, $\tau = 108.32$ kN/m^2 (b) $\sigma' = 317.74$ kN/m^2 $\tau = 177.46$ kN/m^2

(16) A clay sample was consolidated in a triaxial test chamber under an all-around confining pressure of 152 kN/m^2. The sample was then loaded to failure in undrained condition by applying an additional axial stress of 193 kN/m^2. A pore water pressure sensor recorded an excess pore pressure, $(\Delta u_d)_f$ = - 27.6 kN/m^2 at failure. Determine the undrained and drained friction angles for the soil. **Ans. $\phi = 22.9°$ $\phi' = 20.48°$**

(17) The shear strength of a normally consolidated clay can be given by the equation $\tau_f = \sigma \tan 27°$. Following are the results of a consolidated-undrained test on the clay.

- Chamber-confining pressure = 150 kN/m^2
- Deviator stress at failure = 120 kN/m^2

a. Determine the consolidated-undrained friction angle

b. Pore water pressure developed in the specimen at failure

Ans. $\phi = 16.6°$, $(\Delta u_d)_f = 77.8$ kN/m^2

(18) If a consolidated drained test is conducted on the clay specimen of Problem 17 with the same chamber-confining pressure of 150 kN/m^2, what would be the deviator stress at failure? **Ans. 249.4 kN/m^2**

(19) A consolidated-undrained triaxial test was conducted on a dense sand with a chamber confining pressure of 138 kN/m^2. Results showed that $\phi' = 24°$ and $\phi = 31°$. Determine the deviator stress and the pore water pressure at failure. If the sand were loose, what would have been the expected behavior? Explain. **Ans. $\Delta\sigma_{d)f} = 431.12$ kN/m^2 $\Delta u_{d)f} = -30.83$ kN/m^2**

(20) Undisturbed samples from a normally consolidated clay layer were collected during a field exploration program. Drained triaxial tests showed

that the effective friction angle ϕ'=28°. The unconfined compressive strength, q_u, of a similar specimen, was found to be 148 kN/m^2. Determine the pore pressure at failure for the unconfined compression test.

Ans. $\Delta u_{d)f}$ = -83 kN/m^2

(21) Results of two consolidated drained triaxial tests on a clayey soil are as follows:

Test No.	σ'_3 (kN/m^2)	$\sigma'_{1(failure)}$ (kN/m^2)
1	104	320
2	207	517

Find c′ ,ϕ', a, and α **Ans. c′ = 43.55 kN/m^2,ϕ' = 18.26°, a = 42.36 kN/m^2, α = 17.4°**

(22) If the initial stresses is: $\sigma_v > \sigma_h > 0$, find the stress path for the following cases. (a) $\Delta\sigma_v > 0$, $\Delta\sigma_h = 0$ (b) $\Delta\sigma_v = 0$, $\Delta\sigma_h > 0$ (c) $\Delta\sigma_h = -\Delta\sigma_v$ (d) $\Delta\sigma_v = \Delta\sigma_h$ (e) $\Delta\sigma_h = 0.25\ \Delta\sigma_v$

(23) A specimen of normally consolidated clay is consolidated to 207 kN/m^2 and then failed by increasing σ'_1 and σ'_3 and in such a way that $\Delta\sigma_3 = \frac{1}{3}\Delta\sigma_1$

Find q_f and p'_f, knowing that ϕ' = 22° **Ans. q_f = 309.2 kN/m^2 p'_f = 825.4 kN/m^2**

(24) A sample of clayey soil is initially consolidation to 250 kN/m^2, if the relationship in undrained and drained are q = p - 250, q^2+p^2 = 62500 respectively and c´= 53.68kN/m^2, α = 20°. Find σ'_1, σ'_3 and u when

1- q = 40 kN/m^2

2- q at its peak value in drained test

Ans. (1) σ'_1 = 286.78 kN/m^2, σ'_3 = 206.78 kN/m^2, u = 43.22 kN/m^2

(2) σ'_1 = 342.8 kN/m^2, σ'_3 = 86.52 kN/m^2, u = 163.476 kN/m^2

(25) Prove that

$$q_f = \frac{\sigma_{3f} sin\emptyset'}{1 - sin\emptyset'}$$